COMPUTER AIDED GEOMETRICAL VARIATION AND FAIRING OF SHIP HULL FORMS

Frederick Robertson Haberlandt

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS
BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (*and Subtitle*)
COMPUTER AIDED GEOMETRICAL VARIATION AND FAIRING OF SHIP HULL FORMS

5. TYPE OF REPORT & PERIOD COVERED
THESIS

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(*s*)
HABERLANDT, FREDERICK R.

8. CONTRACT OR GRANT NUMBER(*s*)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
MASS. INST. OF TECHNOLOGY

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
CODE 031
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA, 93940

12. REPORT DATE
MAY 78

13. NUMBER OF PAGES
159

14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)

15. SECURITY CLASS. (*of this report*)
UNCLASS

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)
GEOMETRICAL VARIATION; FAIRING OF SHIP HULL FORMS; HULL FORMS

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)
SEE REVERSE.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601 |

UNCLASS
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

T196275

COMPUTER AIDED GEOMETRICAL VARIATION AND FAIRING OF SHIP HULL FORMS

by

FREDERICK ROBERTSON HABERLANDT

Submitted to the Department of Ocean Engineering in May 1978, in partial fulfillment of the requirements for the Degree of Ocean Engineer and the Degree of Master of Science in Naval Architecture and Marine Engineering.

ABSTRACT

Two distinct aspects of computer aided ship design are addressed in this thesis. First, a geometrical hull form modification technique employing the longitudinal repositioning of sections is developed. The second aspect deals with the mathematical representation of lines and line fairing. Before this is done however, justification is presented for utilizing third degree polynomials as an approximation to the spline curves of the naval architect. The results obtained indicate that a fairing procedure based on a least squares curve fitting criteria and a lines representation procedure based on parametric cubic equations could be adapted to generate faired hull forms from the roughest preliminary hull design. Additionally, the hull form modification technique could be programmed so as to produce designs with desired values of C_p, LCB, C_w and LCF from a basis design of similar type.

Thesis Supervisor: Professor Chryssostomos Chryssostomidis
Title: Associate Professor of Naval Architecture

Thesis
H1538
c.1

Approved for public release;
distribution unlimited.

COMPUTER AIDED GEOMETRICAL VARIATION AND FAIRING OF SHIP HULL FORMS

by

FREDERICK ROBERTSON HABERLANDT

B.S., Mechanical Engineering, University of Florida
(1971)

Submitted in Partial Fulfillment
of the Requirements for the
Degree of

OCEAN ENGINEER

and the Degree of

MASTER OF SCIENCE IN NAVAL ARCHITECTURE
AND MARINE ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1978

© Frederick Robertson Haberlandt 1978

COMPUTER AIDED GEOMETRICAL VARIATION AND FAIRING OF SHIP HULL FORMS

by

FREDERICK ROBERTSON HABERLANDT

Submitted to the Department of Ocean Engineering in May 1978, in partial fulfillment of the requirements for the Degree of Ocean Engineer and the Degree of Master of Science in Naval Architecture and Marine Engineering.

ABSTRACT

Two distinct aspects of computer aided ship design are addressed in this thesis. First, a geometrical hull form modification technique employing the longitudinal repositioning of sections is developed. The second aspect deals with the mathematical representation of lines and line fairing. Before this is done however, justification is presented for utilizing third degree polynomials as an approximation to the spline curves of the naval architect. The results obtained indicate that a fairing procedure based on a least squares curve fitting criteria and a lines representation procedure based on parametric cubic equations could be adapted to generate faired hull forms from the roughest preliminary hull design. Additionally, the hull form modification technique could be programmed so as to produce designs with desired values of C_p, LCB, C_w and LCF from a basis design of similar type.

Thesis Supervisor: Professor Chryssostomos Chryssostomidis
Title: Associate Professor of Naval Architecture

ACKNOWLEDGEMENTS

The author is deeply indebted to his advisor, Professor Chryssostomidis for his tireless efforts during the course of this work. A deep debt of gratitude is also owed Professor Heinrich Söding from the University of Hanover, Hanover, West Germany. His contribution of the parametric rotating spline technique added greatly to the capability of the final program.

The author would also like to thank Mrs. Sandy Margeson for her masterful typing of an, all too often, illegible manuscript.

Finally the author wishes to thank his wife and son who, uncomplainingly, sacrificed many weekends and evenings of his companionship through the course of this work.

TABLE OF CONTENTS

Page

TITLE PAGE. 1

ABSTRACT. 2

ACKNOWLEDGEMENTS. 3

TABLE OF CONTENTS 4

LIST OF FIGURES 7

1. Introduction. 9

   1.1 Background. 9
   1.2 Thesis Content. 10

2. Method of Hull Form Modification. 13

   2.1 Background. 13
   2.2 Development 21
      2.2.1 The "One Minus Prismatic" Variation . . . 21
      2.2.2 Varying the Fullness of an Entrance or Run not Associated with Parallel Middlebody. 24
      2.2.3 Modification of Lackenby's Method to Accommodate Hull Forms with Maximum Sections not at Midships. 31
      2.2.4 A Method by Which Constraints may be Placed on the Design Waterline. 33
      2.2.5 A Method by which Constraints may be Placed on the Ship's Profile. 38

3. Mathematical Representation of the Lines of a Ship. 40

   3.1 Background. 40
   3.2 Development 42
      3.2.1 Derivation of the Spline Cubic Equation Using a Variational Approach . . 42
      3.2.2 Deflections Due to Bending of a Simple Elastic Beam. 49
      3.2.3 Piece-wise Continuous Cubic Polynomial Approximation 54
      3.2.4 The Rotating Spline 59

4. Mathematical Fairing of Lines. 62

4.1 Background 62
4.2 Development. .64
    4.2.1 The Least-Squares Criteria for Defining the Cubic Curve. 64
    4.2.2 The Moving Strip Method. 67
        4.2.2.1 STRIP1 68
        4.2.2.2 STRIP2 70
        4.2.2.3 STRIP3 72
        4.2.2.4 TRANS1 75
    4.2.3 Fairing of Curves with Infinite Slope. . .76
        4.2.3.1 Other Transformations. . . . . . .

5. Computer Algorithms. 80

5.1 Overview 80
    5.1.1 Specification of Point Type. 81
    5.1.2 Specification of End Conditions. 83
    5.1.3 Storage of Pertinent Line Data 84
5.2 Description of Subroutines 87
    5.2.1 Lines Fairing. 87
        5.2.1.1 Subroutine PREFAR. 87
        5.2.1.2 Subroutine FARCRV. 88
        5.2.1.3 Subroutine FARLIN. 88
        5.2.1.4 Subroutine FSTPTS. 89
        5.2.1.5 Subroutine TRANS1. 89
        5.2.1.6 Subroutine STRIP1, 2 or 3. 89
    5.2.2 Lines Representation 89
        5.2.2.1 Subroutine PRESPL. 90
        5.2.2.2 Subroutine SPLINE. 90
        5.2.2.3 Subroutine INTERP. 91
        5.2.2.4 Subroutine CALCY 91
        5.2.2.5 Subroutine CALCT 92

6. Comclusions and Recommendations. 93

6.1 Hull Form Modification 93
6.2 Mathematical Representation of Lines and Fairing. 96
    6.2.1 Lines Representation 96
    6.2.2 Fairing. 97
    6.2.3 Recommendations. 97

REFERENCES 103

APPENDIX A. 106
APPENDIX B. 108
APPENDIX C. 112
APPENDIX D. 116
APPENDIX E. 120
APPENDIX F. 123
APPENDIX G. 156

LIST OF FIGURES

2.1 Ship's Lines. 14

2.2 Sectional Area Curve. 17

2.3 Sectional Area Curve (with parallel middlebody) . 21

2.4 Sectional Area Curve. 26

2.5 Sectional Area Curve. 28

2.6 Sectional Area Curve. 31

2.7 Sectional Area and Waterline Curves 35

2.8 Coefficient Ratio Curve 36

3.1 Strained Beam Element 49

3.2 Discretely Loaded Beam. 52

3.3 Spline Curve Fitting Routine. 54

3.4 Rotating Spline Routine 59

4.1 Least Squares Fit 64

4.2 Pinned End. 70

4.3 Clamped End 72

4.4 Axis Rotation 76

5.1 Point Type Examples 82

6.1 Control Lines for a Typical Bulbous Bow Destroyer 99

B.1 Sectional Area Curve. 108

C.1 Section Modification. 112

D.1 Rotating Spline Routine 116

F.1 Flowchart of Fairing Subroutines. 124

F.2 Flowchart of Splinning and Interpolation Subroutines 125

G.1 Bow Section 157

1. Introduction

1.1 Background

Because of the environment in which they operate, the seakeeping characteristics of a ship are of paramount importance when assessing its overall performance. In the past this aspect of a ship's performance had to be judged by the results of model tests conducted at a point in time well into the preliminary design phase. While these tests provide results of good quality, they were not obtained until the pending design was quite firmly established. In fact, the results obtained by model tests had the characteristic of being just that, results, rather than an important input into the design cycle. The obvious desire then would be to have a tool capable of providing accurate predictions of seakeeping performance based on the data available in the conceptual design phase. These predictions could then be used to influence the selection of hull form coefficients, etc. prior to the time when the hull form is actually being generated.

In 1975 Professors T. Loukakis and C. Chryssostomidis published the "Seakeeping Standard Series for Cruiser-Stern Ships".[1] This paper corrolates the seakeeping behavior, as predicted by computer model, of the Extended Series 60

hull forms and sets forth a method by which the performance of this type of ship may be predicted based on five parameters: Froude number, F; ratio of significant wave height to ship length, S; beam/draft ratio, B/T; length/beam ratio, L/B; and block coefficient, CB. With this procedure a designer can predict the relative merits of various candidate designs at a very early stage. This represents a significant capability.

As a result of the work represented in reference [1] there is considerable interest in generating a similar seakeeping series for contemporary cruiser/destroyer type hull forms. In order to do this in the fashion of reference [1], a representative sample of the ship type must be analyzed by computer model and then the results corrolated. It was this need for sample hull forms that provided the motivation for this thesis.

1.2 Thesis Content

There are two aspects of hull form generation addressed in this thesis: first, hull form modification and second, mathematical lines representation and fairing. The technique of hull form modification developed in chapter two is based on the work of H. Lackenby reported in reference [2]. The essence of this method is that the sectional area curve

of an existing ship is redrawn in a systematic fashion to produce a curve with the desired values of prismatic coefficient, C_p, and longitudinal center of buoyancy, LCB. The sections are then shifted longitudinally to produce a modified form with these characteristics. In applying this technique to destroyer type ships there were several anomalies encountered which required that the method of Lackenby be further modified. These modifications, with the pertinent background are contained in chapter two.

The other aspect of lines generation which is addressed in chapters three and four is mathematical lines representation and fairing. Although the fairness of a hull form is not critical to the seakeeping analysis it is an unavoidable subject when considering computer aided ship design. In these chapters the use of parametric cubic splines and least squares curve fitting are addressed. While the parametric splines are shown to provide the capability of representing virtually any type of line, the least squares fairing technique is limited to use with curves representable by single valued functions. The algorithms are, however, capable of fairing lines with infinite slopes at the end points.

It is anticipated that the tools developed in this thesis could be readily fused into a single computer program with the capability of modifying an existing ship form to

obtain a faired design with the desired coefficients of form. When this is developed it will be possible to generate rapidly any number of designs for subsequent performance analysis. The implications of this are discussed in chapter six.

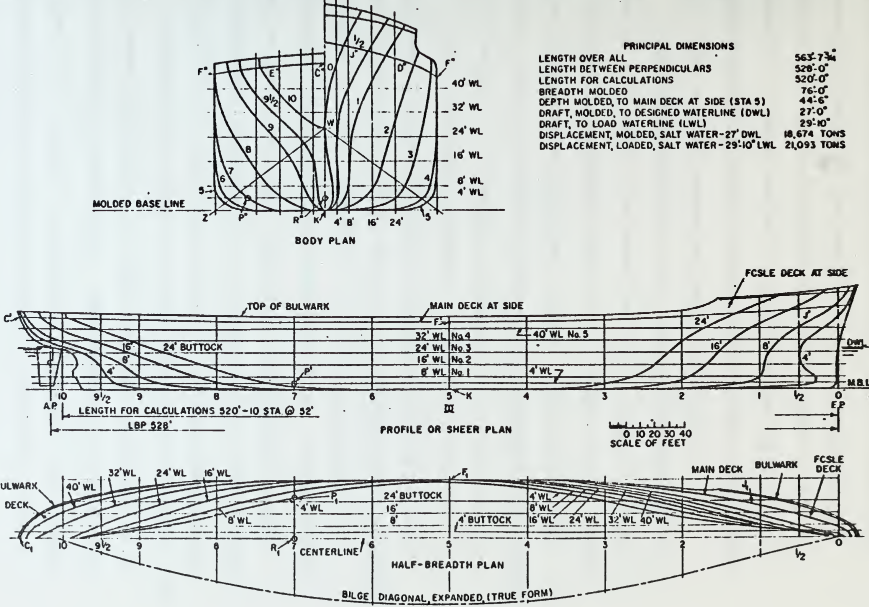
2. Method of Hull Form Modification

2.1 Background

During the design of all but the most trivial engineering systems, it is incumbent upon the engineer that he or she formulate a model of that system. Additionally, the designer must continually refine the model with each successive iterative cycle so that the results are of sufficient detail to be meaningful [3]. One such model used during ship design is a geometric description of the ship's hull form. The most traditional manner of providing this information is by way of the lines drawing.

The ship's lines drawing, more frequently referred to as the ship's lines, is a set of three orthogonal views of the ship's hull depicting the lines of intersection of various planes with the hull form. When viewed in conjunction with one another, they provide the capability to spatially locate any point on the moulded surface of the ship. Figure 2.1, taken from reference [4], is an example of a lines drawing for a "Mariner"-class, steel hull cargo vessel.

While the lines drawing, prepared manually by the naval architect and draftsman, has been the older and more traditional means of depicting the geometric properties of a ship,



PRINCIPAL DIMENSIONS	
LENGTH OVER ALL	563'-7¾"
LENGTH BETWEEN PERPENDICULARS	528'-0"
LENGTH FOR CALCULATIONS	520'-0"
BREADTH MOLDED	76'-0"
DEPTH MOLDED, TO MAIN DECK AT SIDE (STA 5)	44'-6"
DRAFT, MOLDED, TO DESIGNED WATERLINE (DWL)	27'-0"
DRAFT, TO LOAD WATERLINE (LWL)	29'-10"
DISPLACEMENT, MOLDED, SALT WATER-27' DWL	18,674 TONS
DISPLACEMENT, LOADED, SALT WATER-29'-10" LWL	21,093 TONS

Figure 2.1 - Ship's Lines [4]

the advent of the high speed computer has provided great impetus for defining the ship's form in a mathematical format [5]. It is interesting however, that a very successful attempt was made at representing ship's lines mathematically by Admiral David Taylor in the early 1900's [6]. This will be expanded upon a bit later.

When first confronted with the job of creating the lines of a new ship, the naval architect seeks a means of quantifying the expected form of the vessel so that he may strive to create an "optimum" design. These optimizing criteria generally take the form of requirements and restrictions placed on the various coefficients of form, i.e., C_p, C_w, LCB, LCF, etc. However, there might also be requirements placed on certain specific regions of the ship. An example of this could be the shape of the midships section for a cargo vessel or the stern configuration dictated by propeller and rudder selections. Nonetheless, when the naval architect completes his candidate design, the important product will be a faired set of ship's lines meeting all the optimizing criteria previously established.

The above procedure is clearly long and involved. For this reason much effort has been expended to develop hull form modification techniques. The objective of these procedures is to utilize an existing, successful hull design, or parent form, as a basis and then to alter this form in a

systematic fashion. This modified form should have the desired characteristics, and, hopefully, require little additional refairing. It is this fairing procedure, described in chapter 4, which requires a large proportion of the designer's effort. The remainder of this chapter addresses the modification techniques themselves.

One of the oldest and most widely used methods of hull form modification is illustrated in Figure 2.2. In essence, the sectional area curve of an existing ship is altered by some arbitrary or systematic method to produce a curve which satisfies some criteria of the designer, usually prismatic or block coefficient and longitudinal center of buoyancy. The offsets for the new design are then obtained by taking the section in the parent whose ordinate in the sectional area curve matches the ordinate of the derived curve. This is represented by the movement of section A at position X_a in the parent to section A' at position $X_{a'}$ in the derived hull form. Hence, it is merely a longitudinal repositioning of the existing sections. This method works reasonably well as long as the shifts are of "moderate" amount and the designer is prepared to accept the resulting profile and waterlines without alteration.

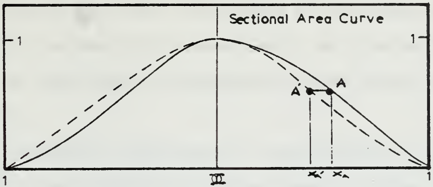


Figure 2.2

The above method lends itself quite well to design without the aid of computers or other automatic computational devices. However, in recent years there has been much work done in the area of hull form modification with the use of high speed digital computers. In virtually all cases where computers are used, an effort is made to represent the ship's contours or surface regions [5] in a mathematical format. It is for this reason that the "Taylor Standard Series" is of interest. It was Admiral David Taylor who, in the early 1900's, generated one of the first successful hull series based on representing the sectional area curve and design waterlines by fifth degree polynomials [6].

Another procedure of hull form modification utilizes specific transformation functions to alter various regions or characteristics of the hull [7]. This form of

modification provides the user with much greater control over the specific ship form than the method of shifting sections longitudinally as previously described. An interesting description of this type of procedure may be found in reference [7].

It is, however, the method of longitudinally shifting sections which was selected for development in this thesis. The reasons for its selection are twofold. First, preliminary work with destroyer-type ships conducted at M.I.T. during the summer of 1977, indicated that the results of the modification were quite realistic and not plagued by gross unfairness. Secondly, the method was tractable and readily adopted to the peculiarities of destroyers. Those peculiarities being principly the fact that this type of ship has no parallel middle body and also that the section of maximum area, in most cases lies at a location other than midship.

The specific method of modification used is that of Lackenby [2] as subsequently modified by Moor [8], and then again by this author. Briefly, the developments presented in reference [2] are highly general, permitting the designer to vary the value of prismatic coefficient, C_p, and the longitudinal center of buoyancy, LCB, of a very wide variety of ships. Included was the capability of altering, or retaining unchanged, the parallel middle bodies of ships so configured. However, one serious drawback was that the method left no

control over the design waterline, and while this line might turn out fair, the longitudinal center of floatation, LCF, merely ended up where it did. It was to this problem that Moor [8] was concerned. By addressing himself to both the sectional area curve and the design waterline in the manner of Lackenby, and then coupling the two procedures, he was able to obtain a derived form having the desired values of C_p, C_w, LCB and LCF.

At this point only one minor problem existed with the method as it stood. For ships with keelrise fore or aft it was possible to obtain unwanted oscillations in the ship's centerline profile. To eliminate these oscillations, this author extended the logic of Moor to include the ship's profile. In so doing, the designer may be assured of a derived form having not only the four desired characteristics and coefficients previously mentioned, but also the desired profile. The only restricting requirement, other than the fact that the changes in C_p and C_w be "moderate", is that for the method to be mathematically rigorous the section of maximum beam, sectional area and local draft must be coincident. If this isn't the case a small ($\approx$1%) unpredictable error, based on the parent hull design and the desired changes is introduced.

All of these relationships are developed in full in the following section. The only other alteration to the methods of Lackenby and Moor was that the procedure had to be capable of accommodating destroyer-type ships whose maximum sections fill other than at midship. This change is also included in the derivations that follow.

2.2 Development

2.2.1 The "One Minus Prismatic" Variation

As a means of introducing the method of longitudinal repositioning of sections, the traditional "one minus prismatic" is first developed. This procedure enables the designer to modify the fineness of a parent form by expanding (creating in ships without), or reducing the region of parallel middle body. It is convenient for this, and the following derivations to refer to Figure 2.3 and the following definitions. It should also be noted that the sectional area curve is normalized with respect to both the value of maximum area and length of the half body.

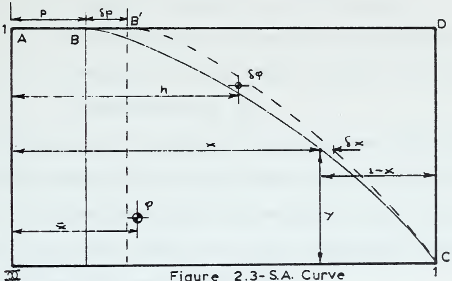


Figure 2.3- S.A. Curve
(with parallel middlebody)

[illegible]

For the parent design:

ϕ = the prismatic coefficient of the half body.

$\bar{x}$ = the fractional distance from midships of the centroids of the half body.

p = the fractional parallel middle of the half body.

x = the fractional distance of any transverse section from midships.

y = normalized area of any transverse section at longitudinal position x.

For the derived form:

$\delta\phi$ = the required change in prismatic coefficient of the half body.

δp = the resulting change in parallel middle body.

δx = the necessary longitudinal shift of the section at x required to generate the required change in prismatic coefficient.

h = the fractional distance from midships of the centroid of the added "sliver" of area represented by $\delta\phi$.

In Figure 2.3 it should be recognized that AB'C is the curve of the derived form and curve ABC is that of the parent. In accordance with the method of the "one minus prismatic" the new location of the transverse sections is defined by the following equations.

$$\frac{1-(x+\delta x)}{1-x} = \frac{1-(\phi+\delta\phi)}{1-\phi}$$

$$\frac{\delta x}{1-x} = \frac{\delta\phi}{1-\phi}$$

$$\delta x = \frac{\delta\phi}{1-\phi}\,(1-x) \qquad (2.1)$$

The area BCD is seen to be $1 - \phi$ and the above modification simply reduces it by the factor $\frac{1 - (\phi+\delta\phi)}{1-\phi}$.

The new area B'CD is therefore $1 - (\phi+\delta\phi)$, demonstrating that the method generates the desired prismatic coefficient of $\phi + \delta\phi$.

There is however a concomitant change in a parallel middle body found by solving for x at $x = p$, i.e.,

$$\delta p = \frac{\delta\phi}{1-\phi}\,(1-p)$$

$$\frac{\delta p}{1-p} = \frac{\delta\phi}{1-\phi} \qquad (2.2)$$

Therefore the resulting change in prismatic coefficient is obtained by altering the length of parallel middle body and then proportionally expanding or contracting the entrance and run. Because of this procedure the method has the following disadvantages:

1. There is no control over the length of the parallel middle body, i.e., ϕ and p, cannot be varied independently.
2. The procedure cannot be applied to reduce the fullness of a ship having no parallel middle body.
3. Conversely, a ship cannot be increased in fullness without introducing parallel middle body.
4. The prismatic coefficient of the entrance or run cannot be altered.
5. The region where fullness is added cannot be controlled. That is, the maximum changes in fullness take place at the shoulders of the curve, i.e., point B.

It is because of these numerous, severe restrictions that Lackenby sought to develop a more general technique of modification.

2.2.2 Varying the Fullness of an Entrance or Run not Associated with Parallel Middle Body.

In reference [2], Lackenby concerned himself with providing a means by which to modify both C_p, LCB and the length of parallel middle body in a controllable manner.

While many of these relationships are of importance, only those which apply more specifically to destroyer-type hull forms will be pursued in any detail. However, for the readers' convenience, the most generalized case of Lackenby's formulas is included as equations (2.10) through (2.13) in the last part of this section. For the in depth derivations the reader is referred to the original paper. Nevertheless, the following derivation is the foundation upon which all the subsequent relationships are based.

In referring to figure 2.4 the various quantities have a meaning identical to those of the previous section. The only additional term requiring definition is k, defined mathematically as follows:

$$k^2 = \frac{1}{3\phi} \int_0^1 x^3 dy$$

The only other difference between figures 2.3 and 2.4 is that figure 2.4 represents a hull form not having parallel middle body, i.e., $p = 0$, and as a consequence the length of the entrance and run equals that of the half length of the ship.

In referring to figure 2.4, it is recognized that in order to preserve the form of the parent at both the end of the ship, $(x = 1)$ and the middle of the ship, $(x = 0)$ an equation for δx of the following form would suffice:

$$\delta x = cx(1 - x)$$

where c is an as yet to be determined constant. It may be seen in Appendix A that the relationship for δx is:

$$\delta x = \frac{\delta\phi}{\phi(1-2\bar{x})} x\ (1 - x) \qquad (2.3)$$

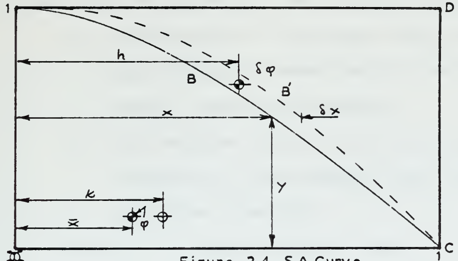


Figure 2.4- S.A.Curve

Clearly, the equation for δx is of the form of a second degree polynomial (parabola) whose values are 0 at $x = 0$ and $x = 1$, as desired. This relationship also shows that the amount by which any section in the parent is shifted is a function solely of the unchanged longitudinal position x and some as yet unknown value $\delta\phi$.

At this point we must turn our attention and consider both the entrance and run concurrently if we are to lend some significance to the quantity $\delta\phi$. If we desire to specify both C_p and LCB, we are in essence placing a requirement on the area under the sectional area curve and its moment about some axis (say $x = 0$). Since equation (2.3) applies independently to the entrance and run, it should be possible to select the $\delta\phi$'s of these respective regions such that when taken together, the ship has the values of C_p and LCB desired.

At this point we introduce the following quantities:

$\bar{z}$ = the distance of the parent ship's LCB from midships, normalized by the half length, (positive forward).

$\delta\bar{z}$ = the required shift in LCB to obtain that required for the daughter hull form (positive forward).

Prime (') - denotes derived forms.

Subscripts - e = entrance or forward half-body.
r = run or after half-body.
t = a property describing the entire ship.

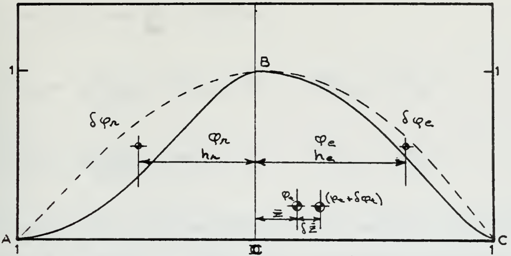


Figure 2.5 - S.A. Curve

Referring to figure 2.5 above, we may interpret the requirements on C_p and LCB mathematically as follows:

$C_p = \phi$

$$\phi_t' = \phi_t + \delta\phi_t = (\phi_e + \phi_r) + (\delta\phi_e + \delta\phi_r) \qquad (2.4)$$

LCB = z

$$z' = \bar{z} + \delta\bar{z}$$

Summing moments of areas

$$z'(\phi_t + \delta\phi_t) = \bar{x}_e\phi_e + h_e\delta\phi_e - [\bar{x}_r\phi_r + h_r\delta\phi_r]$$

$$z' = \frac{1}{\phi_t'} \{\bar{x}_e\phi_e + h_e\delta\phi_e - [\bar{x}_r\phi_r + h_r\delta\phi_r]\} \qquad (2.5)$$

If equations (2.4) and (2.5) are rearranged, the expressions for $\delta\phi_e$ and $\delta\phi_r$ may be obtained.

$$\delta\phi_e = \frac{2}{(h_e+h_r)} \{\delta\phi_t(h_r+\bar{z}) + \delta\bar{z}(\phi_t + \delta\phi_t)\} \qquad (2.6)$$

$$\delta\phi_r = \frac{2}{(h_e+h_r)} \{\delta\phi_t(h_e-\bar{z}) - \delta\bar{z}(\phi_t + \delta\phi_t)\} \qquad (2.7)$$

At this point the only variables which were not previously defined are h_e and h_r. The exact expression for these variables is, with the appropriate subscript:

$$h = \frac{2\bar{x}-3k^2}{1-2\bar{x}} + \frac{\delta\phi}{\phi} \left\{\frac{(\bar{x} - 3k^2 + 2r^3)}{(1 - 2\bar{x})^2}\right\} \qquad (2.8)$$

While equation (2.8) contains $\delta\phi$, the very thing for which it is being used to calculate, it has been stated [2] that the leading term along provides a very good approximation to h for "moderate" values of $\delta\phi$, i.e.,

$$h = \frac{2\bar{x} - 3k^2}{1 - 2\bar{x}} \qquad (2.9)$$

Should it be desired to calculate $\delta\phi$ using equation (2.8), the solution will prove to be a quadratic which, while unwieldy, is certainly not unsolvable. The derivation of h may be found in Appendix A with the value of r defined as follows:

$$r^3 = \frac{1}{4\phi} \int_0^1 x^4 \, dy$$

While the above equations with the derivations in the Appendix illustrate the underlying theory, the following expressions represent the most general form of Lackenby's work.

$$\delta\phi_e = \frac{1}{B_e + B_r} \{2[\delta\phi_t(B_r+\bar{z})+\delta\bar{z}(\phi_t+\delta\phi_t)]+C_e\delta p_e-C_r\delta p_r\} \tag{2.10}$$

$$\delta\phi_r = \frac{1}{B_e + B_r} \{2[\delta\phi_t(B_e-\bar{z})-\delta\bar{z}(\phi_t-\delta\phi_t)]-C_e\delta p_e-C_r\delta p_r\} \tag{2.11}$$

In the following expressions the items refer to the entrance or run as appropriate:

$$\delta x = (1 - x) \{\frac{\delta p}{1-p} + \frac{(x-p)}{A} [\delta\phi-\delta p \frac{(1-\phi)}{(1-p)}]\} \tag{2.12}$$

The practical limits on $\delta\phi_e$ and $\delta\phi_r$ are:

$$\delta\phi = \frac{\delta p(1-\phi) \pm \frac{1}{2}A[1 - \frac{\delta p}{1-p}]}{1-p} \tag{2.13}$$

A, B and C are calculated as follows:

$$A = \phi(1-2\bar{x}) - p(1-\phi) \tag{2.14}$$

$$B = \frac{\phi}{A}\{2\bar{x} - 3k^2 - p(1-2\bar{x})\} \tag{2.15}$$

$$C = \frac{1}{1-p}\{B(1-\phi) - \phi(1-2\bar{x})\} \tag{2.16}$$

It should be recognized that in the above equations the necessary section shift, δx, is a function of the new values of C_p and LCB, and the properties of the original parent hull form.

2.2.3 Modification of Lackenby's Method to Accommodate Hull Forms with Maximum Sections not at Midships.

It should be realized that in all of the preceding developments it was assumed that the section of maximum area fell at midships. While this is certainly the case for a large class of vessels, it is virtually never true for contemporary cruisers and destroyers. It is for this reason that a new set of equations was sought while still adhering to the basic philosophy of longitudinally shifting sections.

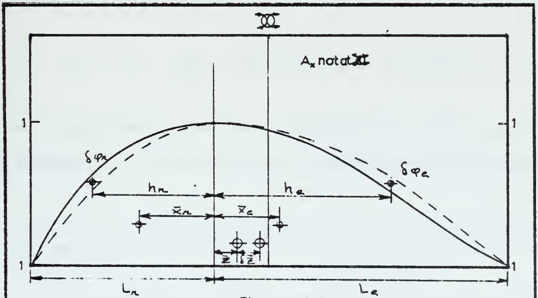


Figure 2.6 - S.A. Curve

Referring to figure 2.6, the definition of terms is, once again, consistant with the preceding section. There are, however, two very important changes. First, where $\overline{z}$ and $\delta\overline{z}$ were originally normalized by the ships half length, they are now normalized by twice that length or the length between perpendiculars, L. Second, x, the local longitudinal position of any section is normalized by the appropriate length of entrance or run L_e or L_r respectively. Also all values of x and $\overline{z}$ take as their origin the station of maximum sectional area.

The basic relationship for δx is still of the form $\delta x = cx(1-x)$ or $\delta x = \frac{\delta\phi}{\phi(1-2x)}\, x(1-x)$, the same as equation (2.3) previously. However, equations (2.4) and (2.5) now become:

$$\phi_t = \frac{1}{L}\{L_e(\phi_e + \delta\phi_e) + L_r(\phi_r + \delta\phi_r)\} \tag{2.17}$$

$$\overline{z}' = \overline{z} + \delta\overline{z} = \frac{1}{L^2\phi_t}\{L_e^2(\phi_e\overline{x}_e+\delta\phi_e h_e)-L_r^2(\phi_r\overline{x}_r+\delta\phi_r h_r)\} \tag{2.18}$$

These two equations are solved simultaneously for $\delta\phi_e$ and $\delta\phi_r$ in Appendix B, the results of which are listed below:

$$\delta\phi_e = \frac{1}{L_e^2h_e+L_eL_rh_r}\{L_r^2\phi_r\overline{x}_r-L_e^2\phi_e\overline{x}_e+\overline{z}'L^2\phi_t'-L_rh_r(L_e\phi_e+L_r\phi_r-L\phi_r')\} \tag{2.19}$$

$$\delta\phi_r = \frac{1}{L_r^2h_r+L_rL_eh_e}\{L_e^2\phi_e\overline{x}_e-L_r^2\phi_r\overline{x}_r-\overline{z}'L^2\phi_t'-L_eh_e(L_e\phi_e+L_r\phi_r-L\phi_t')\} \tag{2.20}$$

It was these two equations along with equation (2.3) which proved to give very satisfactory results for several sample calculations.

2.2.4 A Method by which Constraints may be Placed on the Design Waterline

In the preceding development the designer had no control over the shape of the design waterline. Because the longitudinal center of flotation LCF, was felt to be an important parameter in determining a ship's performance in a seaway, Moor [8] further developed the method of Lackenby to include control over the design waterline. It was with his revised method that Moor and his colleagues developed four distinctly different models with only their midships section identical. They were thus able to cut these four models in half to generate sixteen uniquely different hull forms.

An interesting side light of this experiment was that the parent form used was that of a fast twin-screw currently in service whose maximum section was abaft midships. They, therefore, had to first swing the original area curve to place the maximum section at midships and then proceed with the new method of modification. Although having the maximum section at amidship may have proven to be more tractable for

the creation of the models, it was not necessary for the application of Lackenby's method. This fact was demonstrated in the previous section.

The essence of Moor's method is that the sectional area curve and the design waterline are both altered in the sense of Lackenby to produce the desired values of C_p, LCB, C_w and LCF. At this point a new factor is introduced; the ratio between the sectional area ordinate and the design waterline ordinate is calculated for both the parent and derived hull forms. These ratios are then plotted as a function of ship length and it is from this curve that the longitudinal shift of sections is determined. Figures 2.7 and 2.8 illustrate the sectional area and design waterline curves and the area/waterline ratio curve respectively.

Referring to figure 2.8, to obtain the offsets for a particular section R_d in the derived form, section R_p in the parent is used as a basis. The reason for selecting station R_p in the parent is that it is the closest section to R_d with the same value of area/waterline ratio. The offsets of section R_p are then multiplied by the ratio of the beam coefficient in the derived form at section R_d to that of the parent at section R_p, i.e., B_d/B_p. These values may be seen in figure 2.7. Additionally, if the maximum beam of the derived form is different from that of the parent, the offsets of section R_p also have to be multiplied by the ratio

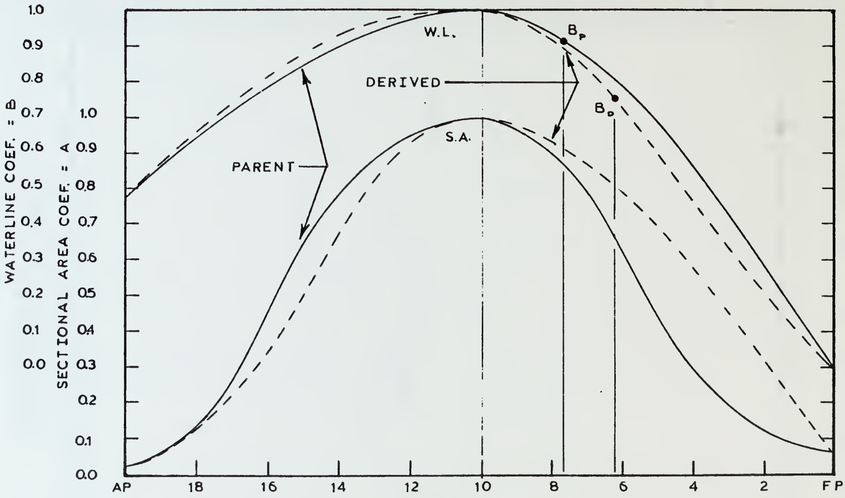


Figure 2.7 — Sectional Area and Waterline Curves.
(Hypothetical)

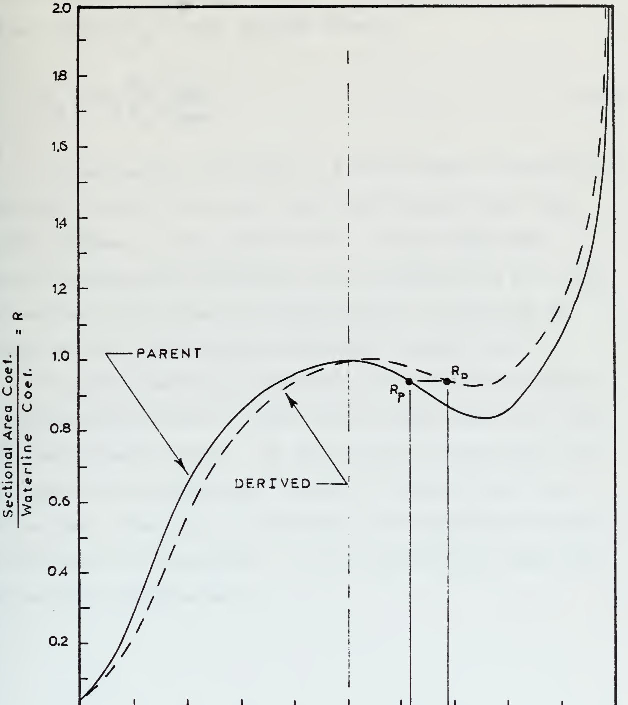


Figure 2.8 – Coefficient Ratio Curve (Hypothetical)

of the maximum beam in the derived form to the maximum beam in the parent, i.e., b_{dmax}/b_{pmax}. Therefore, the equation for any offset b_d in the derived form is:

$$b_d = b_p \frac{B_d}{B_p} \frac{b_{dmax}}{b_{pmax}} \qquad (2.21)$$

It can be seen in figure 2.8 that there are regions of ambiguity. Such is the case where the derived curve lies below a minimum in the parent curve. It has been this author's experience confirming that in reference [8], that the regions which cannot be explicitly be defined may be faired in after defining the sections on either side.

The one remaining undesirable characteristic occurs in regions where there is some form of keel rise, i.e., the fore foot or skeg region. In these areas, if the draft of the parent is proportionally altered to equal that of the derived form, there is a concomitant and undesirable change in the area of the section. It is to this matter which the next section is addressed.

2.2.5 A Method by which Constraints may be Placed on the Ship's Profile

Since Moor's method proved capable of constraining both the sectional area curve and the design waterline, it was decided to extend the method to include the centerline profile of the ship. The actual mechanics require only the introduction of a local draft coefficient, (local draft/ maximum draft) into the denominator of the area/beam ratio. This new ratio, (Area/Beam/Draft), is graphed and the sectional shifts determined from this graph. In determining the new offsets not only are the offsets of the parent modified transversely as described in the previous section, they are also altered in the vertical sense. This alteration is accomplished by using the water plane as a reference and moving the waterlines below a distance proportional to the ratio of the derived form draft coefficient/parent draft coefficient. Also if there is a difference in the maximum draft of the derived form and parent, the waterlines are altered by this ratio as appropriate.

As was mentioned in the background section of this chapter, section 2.1, for this modification technique to be mathematically rigorous the sections of maximum area, maximum design waterline breadth and maximum draft must be coincident. If this is not the case, the actual areas of

the sections generated will be consistantly different by a very small amount from what is desired. From the few examples this author has worked, it is estimated the difference in the value of C_p obtained and that desired is on the order of 1%. The explanation of this is seen in Appendix C.

3. Mathematical Representation of the Lines of a Ship

3.1 Background

It was established in section 2.1 that before the naval architect attempts to actually draw the lines of a new ship, he must have a "firm" description of the new design as represented by the various coefficients and curves of form. Examples of these, as cited previously, are: the sectional area curve, and hence C_p and LCB, the design waterline curve, (C_w and LCF), the principle dimensions and perhaps specific information about the geometry of the midship section or stern region. These characteristics should represent what the designer feels is the "optimum" solution to his set of requirements. The naval architect now has to create one or more, of a possible infinity of, design candidates which fulfill his descriptive coefficients.

The traditional method for drawing the various lines of the ship is with the use of long, continuous strips of wood, metal, or more recently plastic, held in the desired position by weights. These tools are called splines and ducks respectively. The curves produced by this method were continuous but often times contained unwanted waviness. Removing these unwanted undulations, while preserving the

desired character of the line, is a process referred to as fairing. This topic, and the implications of placing a mathematical interpretation on it, are discussed in the next chapter. Not only did the naval architect have to generate smooth curves which pleased him visually, there also had to be a consistancy in location of the surface points when observed from the different views. This is sometimes referred to as cross-fairing and is also addressed in the final chapter. It is the fairing, and cross-fairing, which represents a very large part of the manual design effort.

It was recognized long ago, that if the ship design process was to be automated to any degree, a technique to represent the lines of the ship mathematically would have to be developed. This is especially true today where much of the work is to be done by high speed digital computers. Not only must the designer/programmer provide the mathematical algorithms for representing the ship's lines, he must also provide the logic necessary for the computer to duplicate the heretofore trial and error methods of the draftsman. The alternative to programming the logic however, would be to give the system an interactive man-machine interface at the decision points. It is, however, the mathematical representation of these ships' lines to which this chapter is devoted.

3.2 Development

3.2.1 Derivation of the Spline Cubic Equation using a Variational Approach

There are essentially two different methods by which one may arrive at a mathematical representation for ships lines.

1. Select some mathematical function with several unspecified parameters whose values may be determined by some accuracy criteria and boundary conditions. Typical of this approach is the use of a polynomial and a least squares fit criterion.
2. Choose some smoothness and closeness of fit criteria such that, when taken together with the boundary condition, the function and parameters are determined.

It is this second method, based on a variational smoothness criterion, that will be developed in this chapter.

In general these variational methods involve the minimization of the integral of some linear combination of the squares of the various derivatives of the function sought. In the case where the equation of the flexible spline is sought, the "smoothness" criterion is taken to be the minimization of the strain energy in the spline.

Mathematically this may be represented by [9]:

$$\int_a^b L(y,y',y'',\ldots y^{(n)},x)\, dx = \min \tag{3.1}$$

where, for the spline equation (3.1) becomes:

$$\int_s ck^2 ds \tag{3.2}$$

s = the total path length

c = flexural rigidity of the beam

k = curvature defined mathematically as:

$$= \frac{y''}{(1-y'^2)^{3/2}}$$

ds = elemental arc length

$$= \sqrt{1 + y'^2}\; dx$$

If these values are substituted into equation (3.2) the smoothness criteria becomes:

$$I = \int_{x_1}^{x_m} \frac{y''^2}{(1+y'^2)^{5/2}}\, dx = \min \tag{3.3}$$

To complete the variational problem one must also consider a "closeness of fit" criterion which takes the following form:

$$N = \int_a^b F(y, y', y'', \ldots y^{(n)}, f, x)\, dx \qquad (3.4)$$

It may be seen in most any text on the calculus of variations, e.g., [10] that the criteria for smoothness and closeness of fit may be combined by the introduction of the unknown Lagrange multiplier λ [10]. The results of combining equations (3.1) and (3.4) into a single variational problem is:

$$\delta \int_a^b \{L(y, y', y'', \ldots y^{(n)}, x) +$$

$$\lambda F(y, y', y'', \ldots y^{(n)}, f, x)\}\, dx = 0 \qquad (3.5)$$

A necessary condition for the integral in (3.5) to be stationary is:

$$\frac{\partial (L+\lambda F)}{\partial y} - \frac{d}{dx}\left[\frac{\partial (L+\lambda F)}{\partial y'}\right] + \frac{d^2}{dx^2}\left[\frac{\partial (L+\lambda F)}{\partial y''}\right] -$$

$$\ldots (-1)^n \frac{d^n}{dx^n}\left[\frac{\partial (L+\lambda F)}{\partial y^{(n)}}\right] = 0 \qquad (3.6)$$

with boundary conditions:

$$\Big\{\Big(\frac{\partial L}{\partial y'} - \frac{d}{dx}\frac{\partial L}{\partial y''} + \frac{d^2}{dx^2}\frac{\partial L}{\partial y'''} - \ldots\Big)\delta y + \Big(\frac{\partial L}{\partial y''} - \frac{d}{dx}\frac{\partial L}{\partial y'''} + \frac{d^2}{dx^2}\frac{\partial L}{\partial y^{iv}} -$$

$$\ldots\Big)\delta y' + \Big(\frac{\partial L}{\partial y'''} - \frac{d}{dx}\frac{\partial L}{\partial y^{iv}} + \ldots\Big)\delta y'' + \ldots + \frac{\partial L}{\partial y^{(n)}}\delta y^{(n-1)}\Big\}_a^b = 0 \qquad (3.7)$$

Equation (3.6) is known as the Euler differential equation for the variational problem presented in equation (3.5). The usual procedure for solving this system of equations is to first solve the Euler equation (3.6) in conjunction with the boundary conditions, expressed in equation (3.7). This solution results in an equation of the form $y = f(\lambda,x)$. This equation is then substituted into the "closeness of fit", or accuracy criterion of equation (3.4), to determine the value, or values of λ.

Another aspect of the mechanics of this procedure is revealed when the accuracy criterion requires that the resulting relationship for $y = f(x)$ pass through a discrete set of data points. Such is the case for a "colocative spline", or a spline made to pass exactly through discrete data points. In this instance the integral in equation (3.4) would become a summation. However, in order to preserve the consistancy and similarity of working with integrals in both portions of equation (3.5), the Dirac delta function may be introduced into F.

Redirecting attention to equation (3.3), it will be noted that due to the complexity of the integrand, the differential element of strain energy, the result of equation (3.5) will not be closed form. In order to simplify the above integrand it is assumed that the demonimator, $(1+y'^2)^{5/2}$, is

approximately 1. Or y'^2 is very small. While this may not be the actual case, if the value of y' is linearized as being the value of the slope of the chord between two successive data points, the integral in (3.3) might be thought of as follows:

$$I^* = \int_{x_1}^{x_m} W(x)\, y''^2\, dx = \min \tag{3.8}$$

If we were to take some mean value for W(x) for the entire domain of x, the minimization would be similar to the minimization of:

$$\int_{x_1}^{x_m} y''^2\, dx \tag{3.9}$$

It is also this simplification which will allow us to calculate a closed form solution to $y = f(x)$.

One other simplification which will be made, without harm to generality, is to assume that $x \varepsilon [0,1]$. That is $0 < x_1 < x_2 < \ldots < x_{m-1} < x_m < 1$. At this point we must actually define the accuracy criterion. Assuming that the curve passes exactly through the data points, we may say:

$$y(x_i) = f_i \quad i = 1 \ldots m \tag{3.10}$$

Translating the conditions of (3.9) and (3.10) into the single variational problem of the form of (3.5) we have:

$$\delta \int_0^1 \{y''^2 + 2 \sum_{j=1}^{m} \lambda_j \delta(x-x_j)(f-y)\} = 0 \tag{3.11}$$

It should be recognized that the first delta (to the left of the integral sign) is the symbol for the variation, while the second is the Dirac delta function. f is any "candidate" function passing through the data points $(x_j, y(x_j))$ and the λ_j's are to be determined from the accuracy criteria of equation (3.10).

The resulting Euler equation (3.6) is:

$$y^{iv} - \sum_{j=1}^{m} \lambda_j \delta(x-x_j) = 0 \tag{3.12}$$

The solution of this equation generates a function of the following form:

$$y(\lambda,x) = \frac{1}{12} \sum_{j=1}^{m} \lambda_j |x-x_j|^3 + Ax^3 + Bx^2$$

$$+ Cx + D \tag{3.13}$$

Therefore the value of y and the integration constants A, B, C, and D are all linear functions of the λ_j's.

The boundary equation (3.7) for this specific problem turns out to be of the following form:

$$[-y''' \quad \delta y + y'' \delta y']_0^1 = 0 \qquad (3.14)$$

It must also be noted that (3.13) is valid for $0 \leq x \leq 1$.

While it is not intended to determine equation (3.13) for every possible situation, suffice it to say that the values of the λ_j's and the constants A, B, C, and D are uniquely determined by the coordinate points and the end conditions of the curve [9]. The essential points to be made are:

1. For the criteria used, the equation for the spline as obtained by the variational approach is of the form of a multi-coefficient third degree polynomial.
2. Where the curve is defined over the region (0, 1) by m data points, there is also a need for four additional pieces of information to satisfy, and fully solve equation (3.13).

It will now be demonstrated that the form of equation (3.13) is also supported by the theory developed for the small deflection of elastic beams. As it turns out, the simplifying assumptions made in the preceding derivation are exactly that

which will be made in the small deflection theory. Nevertheless, the consistancy of results lends much reassurance.

3.2.2. Deflections due to Bending of a Simple Elastic Beam

In virtually any undergraduate strength of materials course the subject of beam deflections may be presented in several different ways [11]. However, it will be the method of multiple integration which will be developed here.

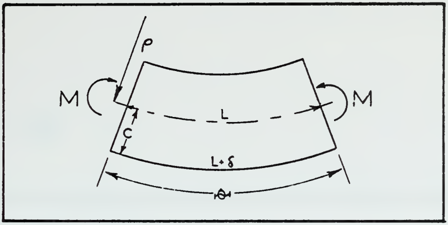


Figure 3.1 - Strained Beam Element (Pure Bending)

Referring to figure 3.1 above, it may be said that the element of the beam deforms about the neutral axis and that as a result, transverse plane sections remain plane after

deformation. This results in an elongation of those fibers outside of the neutral axis and a compression of those fibers inside. Also, the amount of distortion is proportional to the distance from the neutral axis. This being the case, the following equations hold:

$$\theta = \frac{L}{\rho} = \frac{L+\delta}{\rho+c}$$

or, by rearranging,

$$\frac{c}{\rho} = \frac{\rho}{L} = \varepsilon = \frac{\delta}{E} = \frac{M_c}{I}$$

therefore:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad (3.15)$$

In the above equations the following definitions apply:

M = bending moment

E = the stiffness or Young's modulus

I = the moment of inertia of the cross section

ρ = the radius of curvature of the beam, measured to the neutral axis

EI = the "flexural stiffness" or "rigidity"

The mathematical expression for the radius of cruvature, or more traditionally the curvature, K, is defined as follows [12]:

$$\frac{1}{\rho} = K = \frac{d^2y/dx^2}{[1+(\frac{dy}{dx})^2]^{3/2}} \tag{3.16}$$

It is to this equation which the simplification is applied. For actual beams it is assumed that the value of $\frac{dy}{dx}$ is very small, hence, $(\frac{dy}{dx})^2 << 1$. If this is the case, the expression for curvature becomes:

$$\frac{1}{\rho} = K = \frac{d^2y}{dx^2} \tag{3.17}$$

Therefore, when equations (3.15) and (3.17) are combined, the results are as follows:

$$EI\frac{d^2y}{dx^2} = M(x) \tag{3.18}$$

Here, M(x) is meant to indicate that the bending moment is a function of x.

At this point we must address ourselves to the equation for the bending moment in a beam. Specifically, we will look at the results of loading a uniform elastic beam with concentrated point loads; remembering that this case most closely approximates the naval architect's ducks and splines.

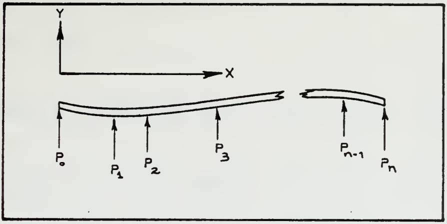


Figure 3.2 - Discretely Loaded Beam

It is advantagious, at this time, to introduce the concept of the singularity function defined as follows:

$$\langle x-x_i \rangle^n = \begin{cases} 0, & x \le x_i \\ (x-x_i)^n & x > x_i \end{cases} \tag{3.19}$$

With the aid of the singularity function, and in reference to figure 3.2, the bending moment equation obtained from the application of concentrated point loads is:

$$\begin{aligned} M(x) &= P_o x + P_1\langle x - x_1\rangle + P_2\langle x - x_2\rangle + \dots \\ &\quad P_n\langle x - x_n\rangle \end{aligned} \tag{3.20}$$

It should be obvious that when equation (3.20) is substituted into equation (3.18) the result may be twice integrated to obtain an equation for $y = f(x)$ which is of the following form:

$$EIy(x) = A + Bx + \frac{1}{6}\{P_o x^3 + P_1 \langle x - x_1\rangle^3 + P_2 \langle x - x_2\rangle^3 \ldots P_n \langle x - x_n\rangle^3\} \qquad (3.21)$$

The above equation is of a form very much the same as equation (3.13). The essential difference is that in equation (3.13), the values of the end forces P_o and P_n were still unknowns requiring the statement of two conditions at each end of the beam. It should also be apparent that the resulting values of the λ_i's are nothing more than 2EI times the forces required to keep the beam in equilibrium. Therefore, based on the results of this and the previous section, it will be accepted without further discussion that the third degree polynomial, or cubic, is an adequate model of the ducks and splines of the naval architect. Hence the term "spline cubic".

3.2.3 Piece-wise Continuous Cubic Polynomial Approximation

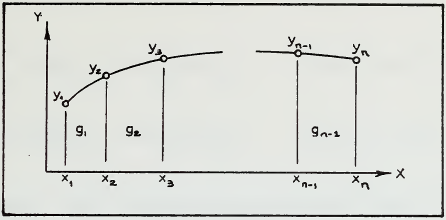


Figure 3.3 - Spline Curve Fitting Routine.

Referring to figure 3.3 above, it is desired to approximate the curve y(x) by some series of cubic functions of the form:

$$g_j(x) = a_j(x-x_j)^3+b_j(x-x_j)^2+c_j(x-x_j)+d_j \qquad (3.22)$$

where j represents the j^{th} interval bounded by x_j and x_{j+1}, and $1 \leq j \leq n - 1$, n being the number of data points. In order for these segments to be continuous we impose the following conditions:

$$(1)\begin{cases} g_j(x_j) = y_j \\ g_{n-1}(x_n) = y_n \end{cases} \qquad j = 1,2,\ldots n-1$$

$$(2)\quad g_j(x_{j+1}) = g_{j+1}(x_{j+1}) \qquad j=1,2,\ldots n-2$$

$$(3)\quad g_j'(x_{j+1}) = g_{j+1}'(x_{j+1}) \qquad j=1,2,\ldots n-2$$

$$(4)\quad g_j''(x_{j+1}) = g_{j+1}''(x_{j+1}) \qquad j=1,2,\ldots n-2$$

From equation (3.22) it should be recognized that to fully describe the curve requires 4(n-1) unknown coefficients. However, the above equations provide 4n-2 conditions. We therefore require two more conditions. The obvious choice for these two additional constraints would be to specify the end conditions for the beam. Specifically, you would specify either g' or g" at the ends.

For the sake of brevity the remainder of this derivation will be abridged to include only the essential equations. For a complete and detailed description of this procedure the reader is referred to references [13] and [14].

Continuing with the derivation, the following definitions will prove useful.

$$h_j = x_{j+1} - x_j \qquad (3.23)$$

$$D_j = (y_{j+1} - y_j)/h_j \tag{3.24}$$

We may now relate the unknown coefficients in the following manner:

$$a_j = \frac{1}{6h_j}(s_{j+1} - s_j) \tag{3.25}$$

$$b_j = \frac{s_j}{2} \tag{3.26}$$

$$c_j = D_j - \frac{h_j}{6}(2s_j + s_{j+1}) \tag{3.27}$$

$$d_j = y_j \tag{3.28}$$

Substituting these equations into condition (3) will generate a relationship between successive values of s_j of the following form:

$$s_j h_j + 2(h_j + h_{j+1})\, s_{j+1} + s_{j+2}\, h_{j+1} = 6(D_{j+1} - D_j)$$

$$j = 1,2,\ldots n-2 \tag{3.29}$$

Equation (3.29) will thus generate the following system of equations:

$$\begin{bmatrix} 2(h_1+h_2) & h_2 & 0 & \cdots & & \\ h_2 & 2(h_2+h_3) & h_3 & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ & h_{n-3} & 2(h_{n-3}+h_{n-2}) & h_{n-2} & & \\ & & h_{n-2} & 2(h_{n-2}+h_{n-1}) & & \\ & & & & & \end{bmatrix} \begin{bmatrix} s_2 \\ s_3 \\ \cdot \\ \cdot \\ \cdot \\ \\ s_{n-1} \\ \\ \end{bmatrix} = \begin{bmatrix} 6(D_2-D_1)-s_1h_1 \\ 6(D_3-D_2) \\ \cdot \\ \cdot \\ \cdot \\ \\ 6(D_{n-1}-D_{n-2})- \\ h_{n-1}s_n \end{bmatrix} \tag{3.30}$$

It can be seen that, for any point x_j, s_j is the curvature at that point. For the above system of equations if the curvature is known at the end points, i.e., s_1 and s_n, the curve will be completely defined.

If instead of curvature the slope is specified at the end point, the above matrix will be modified slightly. In this case, the value of the curvature at the end point will be unknown. For the situation where the beginning slope is specified the following changes will occur. From equation (3.27):

$$D_1 - \frac{h_1}{6}(2s_1 + s_2) = c_1 = t_1$$

or $$2s_1h_1 + s_2h_1 = 6(D_1 - t_1) \tag{3.31}$$

The effect on the matrix system will be to change the currently existing top row and then add another top row and left column as follows:

$$\begin{bmatrix} 2h_1 & h_1 & \cdots & \\ h_1 & 2(h_1+h_2) & h_3 & \cdots \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 6(D_1-t_1) \\ 6(D_2-D_1) \end{bmatrix}$$

If the end slope is specified a change of similar form takes place only adding a row to the bottom and a column to the right side as follows:

$$\begin{bmatrix} \cdots h_{n-2} & 2(h_{n-2}+h_{n-1} & h_{n-1} \\ \cdots & h_{n-1} & 2h_{n-1} \end{bmatrix} \begin{bmatrix} s_{n-1} \\ s_n \end{bmatrix} = \begin{bmatrix} 6(D_{n-1}-D_{n-2}) \\ 6(-D_{n-1}+t_n) \end{bmatrix}$$

The form of the above matrix is tridiagonal and lends itself to rapid solution by a recursive relationship [14]. This fact will save a significant amount of computational time when reduced to a computer algorithm.

3.2.4 The Rotating Spline [15]

One particular disadvantage of the "piece-wise cubic" method developed in section 3.2.3 is that it will not provide a solution for curves having infinite slopes. It is for this reason that method of the "rotating spline" was developed. Referring to figure 3.4 it may be related that this procedure is merely a modification of the "piece-wise cubic" technique.

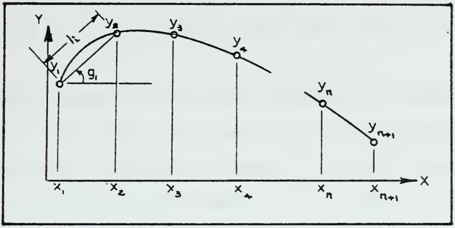


Figure 3.4 - Rotating Spline Routine

For this method of curve approximation a cubic polynomial is generated for each interval as before. However, in this case the coordinate system is redefined for each

interval, and the cubic equation is generated with respect to this local coordinate system.

Appendix E contains the steps necessary to generate the computer algorithm. There is, however, one definate disadvantage to using a rotated coordinate system. In order to obtain an interpolated ordinate on the curve the following two parametric equations must be used.

$$x = x_i+(x_{i+1}-x_i)t-(y_{i+1}-y_i)t(1-t)[a_i(1-t)-b_it] \tag{3.32}$$

and

$$y = y_i+(y_{i+1}-y_i)t+(x_{i+1}-x_i)t(1-t)[a_i(1-t)-b_it] \tag{3.33}$$

Both of these equations are third degree polynomials in the parameter t. To solve for some value y of the point (x,y) in the unrotated coordinate system, x is used in equation (3.32) to solve for t such that $0 \leq t \leq 1$ and t is also real. This value of t is then used in equation (3.33) to calculate y. It should be pointed out that the quantities a_i and b_i were determined previously as described in Appendix E.

In summary, we have shown by two methods, variational calculus and simple beam theory, that the third degree or cubic polynomial provides a good representation of the thin elastic spline used in drawing ships lines. There was,

however, the disadvantage that the equations were not capable of representing curves having infinite slopes. For this reason the method of the rotating spline was introduced. This parametric method permits the representation of virtually any continuous curve, including those which are non singular.

4. Mathematical Fairing of Lines

4.1 Background

It has been stated previously that the lines fairing process can be the most time consuming aspect of the ship design cycle. For this very reason fairing becomes a prime candidate for automation. The difficulty, however, lies in the fact that obtaining a universally accepted mathematical definition of a faired line, or the fairing process itself, is a virtual impossibility. Perhaps the most general definition, and one which would prove the least restrictive, is the following:

> A faired line is one which retains the desired "character" but eliminates any undesired waviness or fluctuations.

It will be shown in the following sections that this result may be achieved by fitting, in a least squares sense, a third degree polynomial to a set of four or five data points. The number being dependent upon the desired boundary conditions. In addition to the similarity of the third degree polynomial to the form of an elastic spline, the polynomial also provides the capability of introducing a desired inflection point into a series of data points. It

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

also prohibits the introduction of multiple inflection points and undesired waviness, also an asset. Because of these characteristics and the excellent results demonstrated in reference [16], this "least squares" criteria was employed as the foundation of the fairing process.

4.2 Development

4.2.1 The Least-Squares Criteria for Defining the Cubic Curve

It was established in chapter three that the cubic polynomial would provide a "good" approximation of the shape of a spline used to construct the lines of a ship. What remains to be shown is how these polynomials are applied in order to generate the faired position of a set of data points.

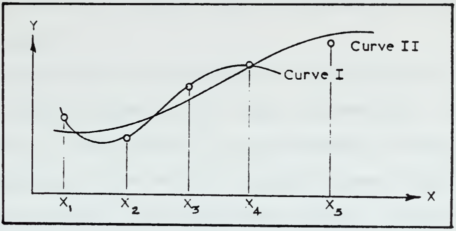
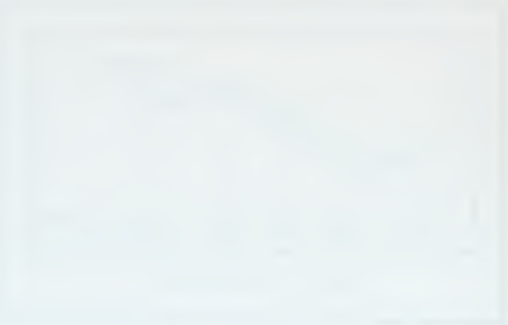


Figure 4.1 - Least Squares Fit

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \qquad (4.1)$$

[illegible]



[illegible]

Referring to equation (4.1) above it should be recognized that, in order to uniquely specify the cubic polynomial in its general form, four independent pieces of information are required. The result of applying this information is to determine the values of a_0, a_1, a_2 and a_3. This will produce a curve which exactly conforms to the given requirements. This is illustrated as curve I in figure 4.1 where the curve is required to pass exactly through the first four data points. The disadvantage of using this type of curve is that, since it is required to pass exactly through the given data points, it is unable to modify their position. It is this alteration however that is necessary if the curve is to be "faired".

As stated above, four pieces of information are required to uniquely specify the cubic polynomial. If, however, we were to over specify the requirements of the curve and then demand that the solution satisfy these requirements in some "best possible" but not exact manner, we begin to get a feel for how fairing can be produced. Mathematically this can be stated as follows.

Referring to curve II in figure 4.1 we will require that our resulting curve pass as close as possible to the five given data points. Usually this translates into a

mathematical form by requiring that the sum of the squares of the distances between the curve and the given data points should be minimized.

$$s = \sum_{i=1}^{5} [y_i - (a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3)]^2 \tag{4.2}$$

or

$$\frac{\partial s}{\partial a_0} = \frac{\partial s}{\partial a_1} \cdots = \frac{\partial s}{\partial a_3} = 0$$

These derivatives generate the following system of normal equations which can be solved for $a_0 \ldots a_3$.

$$\begin{bmatrix} s_0 & s_1 & s_2 & s_3 \\ s_1 & s_2 & s_3 & s_4 \\ s_2 & s_3 & s_4 & s_5 \\ s_3 & s_4 & s_5 & s_6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} t_0 \\ t_1 \\ t_2 \\ t_3 \end{bmatrix} \tag{4.3}$$

Where s_k and t_k are defined as follows:

$$s_k = \sum_{i=1}^{5} x_i^k \tag{4.4}$$

$$t_k = \sum_{i=1}^{5} y_i x_i^k \tag{4.5}$$

It can be seen that curve II in figure 4.1, while not passing exactly through the data points, passes "fairly close" and also displays a smooth and continuous character. It is this closeness of proximity, or minimization of the least squares difference, procedure that is the essence of the fairing criteria used in this thesis.

The following sections will develop the equations for line segments whose end position and slope or just end position are fixed. First, however, a brief explaination of how these segments are applied to fair a complete line or set of data points.

4.2.2 The Moving Strip Method

In the procedure described above it was seen that a least squares spline was passed through five data points and the points on that line were then considered to be fair. In order to fair a complete set of initially unfair points consider only five points at a time, i.e., P_k to P_{k+4}. After passing a least squares spline through these five points obtain the faired position of point P_{k+2}. Then move

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

the strip one unit (k = k+1) and consider the next five points, P_k to P_{k+4}, fairing P_{k+2}. For each step we could use previously faired values for P_k and P_{k+1} expecting our final solution to be obtained more rapidly. By walking this strip of five points through the entire set of data the faired position of each point may be obtained. This procedure may also be found in references [16, 17].

The following three sections develop the equations needed when considering data points whose boundary conditions are of the following type:

1. Free end--the position and slope of the end is unspecified.
2. Pinned end--the end position is fixed but free to rotate.
3. Clamped end--end position and slope are fixed.

4.2.2.1 STRIP1: Fairing an interval with free ends.

This procedure is the same as that developed in section 4.2.1. It is this routine which is used to fair the center data point (P_{k+2}) of five interior data points, i.e., $P_k \neq P_1$ and $P_{k+4} \neq P_n$, where P and P_n are the first and last points respectively in the set of given data. This routine is also used to fair P_1, P_2, P_{n-1} and P_n for the case where the ends are free to both rotate and translate.

Repeating equations (4.3) to (4.5) for convenience.

$$\begin{bmatrix} s_0 & s_1 & s_2 & s_3 \\ s_1 & s_2 & s_3 & s_4 \\ s_2 & s_3 & s_4 & s_5 \\ s_3 & s_4 & s_5 & s_6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_1 \\ t_2 \\ t_3 \end{bmatrix} \tag{4.3}$$

and

$$s_k = \sum_{i=1}^{5} x_i^k \tag{4.4}$$

$$t_k = \sum_{i=1}^{5} y_i x_i^k \tag{4.5}$$

also

$$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \tag{4.1}$$

Therefore the faired position of the second and third points in the five point strip are:

$$\bar{y}_2 = P(x_2) \text{ and } \bar{y}_3 = P(x_3)$$

For the case of a free end point the first point becomes:

$$\bar{y}_1 = P(x_1)$$

It would be fair to expect that the equation P(x) would be most representative of the actual curve at its interior regions where uncertainty about end conditions would have less effect. For this reason only the center point of a strip is recalculated as being faired, i.e., P_{k+2} as opposed to recalculating both P_k and P_{k+1}.

4.2.2.2 STRIP2: Fairing an interval with pinned end.

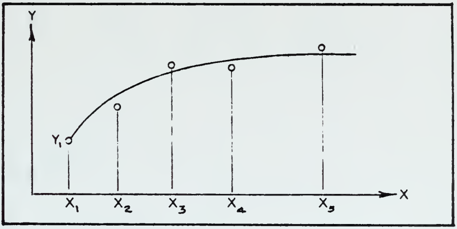
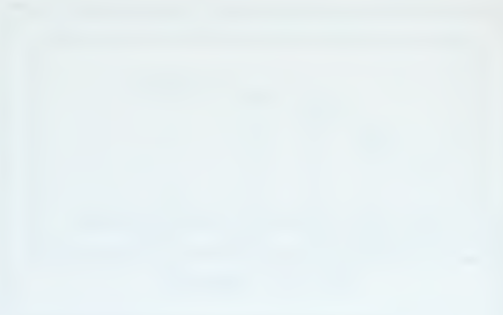


Figure 4.2 - Pinned End



In fitting the curve of equation (4.1) to the five data points in figure 4.2 above, we require that $P(x_1) = y_1$ exactly. If in our calculations we adjust the abscissas such that $x_1 = 0$ we may simplify this equation to:

$$P(x) = y_1 + a_1x + a_2x^2 + a_3x^3 \tag{4.6}$$

Applying our least squares criteria to this we obtain the following normal equations:

$$\begin{bmatrix} s_2 & s_3 & s_4 \\ s_3 & s_4 & s_5 \\ s_4 & s_5 & s_6 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \tag{4.7}$$

where

$$s_k = \sum_{i=2}^{5} x_i'^{k} \tag{4.8}$$

$$t_k = \sum_{i=2}^{5} (y_i - y_1)x_i'^{k} \tag{4.9}$$

and

$$x_i = x_i - x_1$$

With the values of a_1, a_2 and a_3 computed from equation (4.7) we can calculate the faired position of points P_2 and P_3:

$$\bar{y}_2 = P(x_2'),\ \bar{y}_3 = P(x_3')$$

where

$$x'_2 = x_2 - x_1,\ x_3' = x_3 - x_1$$

After these two points are determined STRIP1 can be applied to continue the fairing process

4.2.2.3 STRIP3: Fairing an interval with clamped ends.

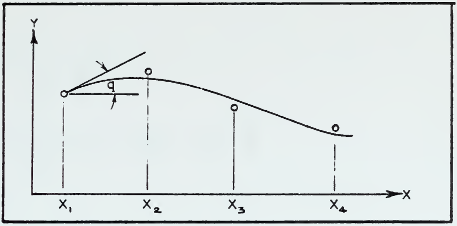


Figure 4.3 – Clamped End

In order to fair an interval with both position and slope of the first point fixed we will consider only the first four points as shown in figure 4.3 above. In order to simplify the derivation we once again adjust the abscissas such that $x_1 = 0$. For this case equation (4.1) becomes:

$$P(x) = y_1 + qx + a_2x^2 + a_3x^3 \tag{4.10}$$

After applying the least squares fit criteria to the four data points the normal equations obtained are:

$$\begin{bmatrix} s_4 & s_5 \\ s_5 & s_6 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} t_2 \\ t_3 \end{bmatrix} \tag{4.11}$$

where

$$s_k = \sum_{i=2}^{4} x_i'^{k} \tag{4.12}$$

$$t_k = \sum_{i=2}^{4} (y_i - qx_i' - y_2)\, x_i^k \tag{4.13}$$

and

$$x_i' = x_i - x_1$$

For the simple 2 X 2 system above a_2 and a_3 may be written as follows:

$$a_2 = \frac{t_2 s_6 - t_3 s_5}{s_4 s_6 - s_5^2} \tag{4.14}$$

$$a_3 = \frac{t_3 s_4 - t_2 s_5}{s_4 s_6 - s_5^2} \tag{4.15}$$

where

$$\Delta = \begin{vmatrix} s_4 & s_5 \\ s_5 & s_6 \end{vmatrix} \neq 0$$

Using the values of a_2 and a_3 calculated the faired position of the second point may be readily determined as:

$$\bar{y}_2 = P(x_2')$$

where

$$x_2' = x_2 - x_1$$

After fairing the second point STRIP1 can be applied to continue the fairing process, fairing point three and four the first time it is applied.

4.2.2.4 TRANS1: Fairing the last points in a given sequence of data.

As can be seen in the previous sections, the fairing procedures operate on a series of points with monotonically increasing abscissa and end conditions specified at x_1; where $x_1 < x_2 < x_3 \ldots$. At the time when the five point fairing interval reaches the other end of the curve, i.e., $P_k = P_{n-4}$, the following transformation must take place:

$$\begin{aligned} x_1' &= x_n - x_n = 0 & y_1' &= y_n \\ x_2' &= x_n - x_{n-1} & y_2' &= y_{n-1} \\ x_3' &= x_n - x_{n-2} & y_3' &= y_{n-2} \\ x_4' &= x_n - x_{n-3} & y_4' &= y_{n-3} \\ x_5' &= x_n - x_{n-4} & y_5' &= y_{n-4} \end{aligned} \qquad (4.16)$$

This allows the fairing of the last three points as if they were the first three points. Once these three points are faired the reverse transformation is employed to place y_1' into y_n, etc.

4.2.3 Fairing of Curves with Infinite Slopes

In ship design it is not infrequent that lines are encountered which possess infinite slopes at one or both end points. Such is the case of a section through the bow of a ship equiped with a bulbous bow. Here if the offsets y are expressed as a function of z, an infinite slope will occur at the bottom of the bulb. While the parametric rotating spline of section 3.2.4 will accommodate such a form, the simple cubic polynomial of equation (4.1) will prove indeterminate for an interval containing an end point with infinite slope.

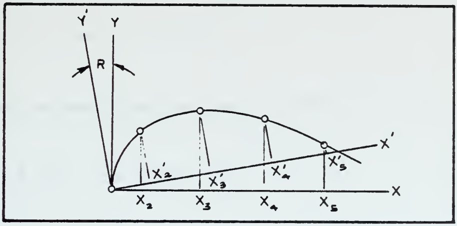


Figure 4.4 - Axis Rotation

Referring to figure 4.4 above, it can be seen that if the axis are rotated by some small anle θ, and the data points are redefined in this new coordinate system the fairing process may be carried out as normal. The following equations relate the coordinates in the two coordinate systems.

$$\left.\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}\right\} \tag{4.17}$$

$$\left.\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}\right\} \tag{4.18}$$

It is also assumed that for small values of θ, e.g., 10°:

$$\Delta y \cong \Delta y', \quad \Delta x \cong \Delta x' = 0 \tag{4.19}$$

In order to continue the fairing process the faired position of the first three points and the slope at the third point could be calculated in the rotated coordinate

system and then transformed back into the unrotated coordinate system. The faired position and slope of the third point could be used to continue the fairing in the unrotated plane. The following slope transformation is also helpful.

$$\frac{dy}{dx} = \tan \{\text{arc} \tan\left(\frac{dy'}{dx'}\right) + \theta\} \qquad (4.20)$$

4.2.3.1 Other transformations.

There are any number of transformations one could use to accommodate the problem of the infinite slope. One tried by this author was that of letting [18]:

$$\left.\begin{array}{ll} x = \frac{1}{2}(1 - \cos\theta) & \\ \text{or} & 0 \leq x \leq 1 \\ \theta = \text{arcos}\ (1 - 2x) & \end{array}\right\} \qquad (4.21)$$

The curve is then plotted as a cubic in θ. This has the advantage of eliminating an infinite slope at $x = 0$; in fact $dy/d\theta \cong \sqrt{r/2}$, where r is the radius of curvature of $y = f(x)$ at $x = 0$. The disadvantage, as seen by this author, is that fairing will take place in the distorted y,θ plane. Additionally, even though the resulting curves appear to be aesthetically pleasing, some apprehension exists regarding the use

of lines faired in the two coordinate planes. For this reason the author opted for fairing in a rotated coordinate system as opposed to one which was distorted.

5. Computer Algorithms

5.1 Overview

The system of subroutines developed in this thesis were designed to provide two distinct capabilities: (1) to provide a means of fairing a series of data points not previously considered fair, and (2) to provide the capability of representing a series of data points by an analytical mathematical expression. This second feature would also provide a means by which slopes, curvatures, etc. could be determined by interpolation. The theory of these two procedures was developed in chapters four and three respectively.

The ultimate objective of these subroutines would be their utilization in a program to fair and draw an entire ship form. Because of this and the virtually infinite nature of the lines existing in a hull form, the program has to be capable of handling many line types. As an example of this, see figure 5.1, the programs require the following information as input.

1. Independent variable coordinates.
2. Dependent variable coordinates.
3. Data point type.
4. Number of data points.

5. Type of end conditions.
6. End slopes if required.
7. An indication as to whether the input data is fair as submitted.

The only other pieces of information required for fairing are:

1. TOL: Is a tolerance representing a limiting distance which any data point may be moved in the fairing process.
2. ACC: This number represents an accuracy which, if during the fairing process a point is not moved by more than this amount, it is considered to be in a faired position.
3. LIMIT: This number sets a limit on the number of iterative cycles permitted in the fairing process.

5.1.1 Specification of Point type

The designation of point type is designed to be as consistant as possible with reference [19].

POINT TYPE	DEFINITION
0	Normal point at the beginning or in the interior of a continuous curved line segment.
1	Break point at the end of a continuous curved segment. At present this point is treated as if it were pinned. The slope, while unspecified, is discontinuous.

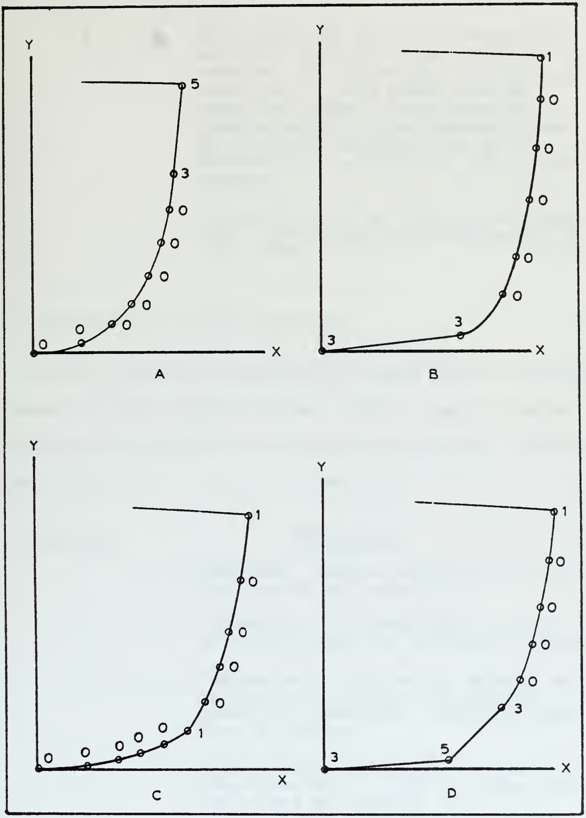


Figure 5.1– Point Type Examples

3	This point can be at the beginning, middle or end of a straight line segment. The slope is continuous at this point. This point must be specified where a curved segment joins a straight line segment since the point is considered to be a clamped end condition for the curved segment.
5	Break point at the end of a straight line. The slope is discontinuous at this point.

5.1.2 Specification of End Conditions

The end condition designation is made with a two digit real number of the following format, "B.E". Here B corresponds to the end conditions at the beginning of the line and E the end condition at the end of the line.

END TYPE	DEFINITION
1	Free end. The end point is free to both rotate and translate.
2	Pinned end. The end point is free to rotate only, the position is fixed.
3	Clamped end. The end is totally constrained. It is free to neither rotate or translate. The slope must also be defined.
4	Clamped end with infinite slope. The slope, whether $\pm \infty$ is determined by the second data point, i.e., $+ \infty$ if $y_2 > y_1$ or $- \infty$ if $y_2 < y_1$.

5.1.3 Storage of Pertinent Line Data

The information necessary to fully describe any line is stored in a 32 x 7 two-dimensional array. This array is labeled CRV in the subroutines and its elements have the following significance. At present the first thirty rows are for data point or interval information and the last two rows are for overall curve characteristics. This could be easily expanded to allow more input data.

1. Colume 1, CRV(1,1) to CRV(30,1):
   Abscissa of the input data points.
2. Colume 2, CRV(1,2) to CRV(30,2):
   Ordinates of the original data points.
3. Column 3, CRV(1,3) to CRV(30,3):
   The faired ordinates of the data points.
4. Column 4, CRV(1,4) to CRV(30,4):
   The point type, see section 5.1.1.
5. Column 5, CRV(1,5) to CRV(29,5):
   The values of a_i as defined in Appendix D.
6. Column 6, CRV(1,6) to CRV(29,6):
   The values of b_i as defined in Appendix D.
7. Column 7, CRV(1,7) to CRV(30,7):
   The slope of the curve at the data points as defined in Appendix D.

It should be noted that the elements of columns 5, 6 and 7 are obtained as a result of the splinning option. The data in column 3 are obtained as a result of exercising the fairing option.

ELEMENT	DEFINITION
CRV(31,2)	The number of data points. At present, $6 \leq CRV(31,2) \leq 30$.
CRV(31,3)	The slope at the beginning of the curve. Left blank if not specified.
CRV(32,1)	An indication of the fairness of the curve. 1.=the data submitted is fair. 2.=the data submitted is not fair.
CRV(32,2)	End condition specification, see section 5.1.2.
CRV(32,3)	The slope at the end of the curve. Left blank if not specified.

The other elements of the CRV matrix are reserved for future use, e.g., in a full ship fairing program.

There are three other aspects of the program which are of the utmost importance. As was seen in the development of chapter three, the mathematical curve representation, or splinning procedure is fully capable of accommodating multi-valued curves. The fairing option, however, requires that a curve be single valued over the domain of the independent variable. Therefore, while it is possible to fit a cubic curve to virtually any series of data points, care must be observed when exercising the fairing option. The second inviolable characteristic of the program is that the data points must be submitted in a monotonically sequential fashion, i.e., the points must not be submitted in a random fashion,

but rather as they are encountered while following the path of the curve. The last consideration is that in order to fair any curved line segment there must be at least six data points in the continuous curved region. This is true regardless of the end conditions of the line as a whole or the end conditions for a line segment.

5.2 Description of Subroutines

A flow chart and subroutine listing may be found in Appendix F.

5.2.1 Lines Fairing

The subroutines included in this section are utilized to calculate the faired position of the given data points, i.e., column 3 of the CRV matrix.

5.2.1.1 Subroutine PREFAR

This subroutine takes the data in the CRV matrix and loads all the points on a continuous curve segment into three linear arrays; X( ), Y( ) and YORIG ( ) representing the abscissa, faired ordinate (the original ordinate for the first iteration) and the original ordinate respectively. This process is governed by the value of point type, CRV(I,4). With these arrays established PREFAR calls either FARCRV or FARLIN, depending on whether the curve segment begins with an infinite slope.

Upon final return to this subroutine the values of the faired position of the data points will have been calculated and placed in column three of the CRV matrix.

5.2.1.2 Subroutine FARCRV

This subroutine takes the data in the X, Y, YORIG array, for those line segments which have infinite slopes, rotates the coordinate axis 10°($\pi/18$ radians) and then places the transformed points, equation (4.17), in an XPRIM, YPRIM and YOPRIM array. The subroutine then calls subroutine FARLIN to fair the first six data points in the rotated system. At this time subroutine SPLINE is called to determine the slope at the third point, also in the rotated system. The subroutine then completes fairing the remaining data points by matching the position and slope at the third point, in the unrotated system. That is, assuming point three to be clamped and beginning with STRIP3.

5.2.1.3 Subroutine FARLIN

This subroutine takes the points of a continuous curve segment and calls the various STRIP_ subroutines which actually compute the faired position of the points. FARLIN also calls subroutines FSTPTS and TRANS1 to fair the first and last points in the sequence.

[illegible]

[illegible]

[illegible]

[illegible]

5.2.1.4 Subroutine FSTPTS

This subroutine fairs the first three data points in a sequence of data points based on the end condition. STRIP1, 2 or 3 are called as appropriate.

5.2.1.5 Subroutine TRANS1

This subroutine fairs the last three data points based on the end condition specified. Specifically it transforms the abscissa in accordance with equation (4.16).

5.2.1.6 Subroutine STRIP1, 2 or 3

These subroutines are described in detail in sections 4.2.2.1 to 4.2.2.3. They use, as arguments, the variables in the X1, Y1 and Y0 arrays. Additionally they require values of TOL and ACC which place limits on the amount which a point may be moved and the amount of movement which is considered to be negligable. For the case where the point would move by more than TOL from its original (unfaired) position, its movement is limited by the value of TOL.

5.2.2 Lines Representation

The methodology of representing a line by a parametric cubic equation was developed in chapter three. The actual

sequence in which the process is executed is described in the following sections.

5.2.2.1 Subroutine PRESPL

This subroutine examines the input data in the CRV matrix and places elements of continuous curved line segments into the X, Y and YORIG arrays. This assignment is based on data point type found in column four of the CRV matrix. Referring to figure 5.1A, the program would load the first eight points into X, Y and YORIG. Point nine, point type 5, would be used in conjunction with point eight to determine the slope of the curved segment ending at point eight. The program then calls SPLINE to carry out the actual curve fitting algorithm.

5.2.2.2 Subroutine SPLINE

This subroutine uses the data in X, Y and YORIG obtained from PRESPL and carries out the curve fitting algorithm presented in Appendix D. Although many intermediate terms are calculated, the only terms which are retained are a_i, b_i and d_i, these quantities are subsequently used to calculate interpolated values of the independent variable and slope at a point specified by the user.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

5.2.2.3 Subroutine INTERP

This subroutine determines the interval in which a desired value of a dependent variable is located. It then passes the coordinates of the surrounding points and the values of a_i and b_i for the interval to subroutine CALCY which calculates the value of the dependent variable and slope at the desired point.

5.2.2.4 Subroutine CALCY

This subroutine calls CALCT to obtain the value of the parametric variable T. With the value of T the interpolated value of the independent variable is determined. Since the dependent and independent variables are represented parametrically, the slope of the curve is calculated by the chain rule as follows:

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} \tag{5.1}$$

Since the value of T is determined as being the root of a third degree polynomial, CALCY is designed to calculate the interpolated value of the independent variable and slope for up to three unique and real values of T. However, the subroutine is designed to print a warning that additional points are needed to specify the curve if T has more than one real value.

5.2.2.5 Subroutine CALCT

This subroutine calculates the roots of the third degree parametric polynomial in T using the algorithm in reference [20]. This procedure is also presented in Appendix E for the readers' convenience. It should be realized, however, that only the real roots are calculated in the subroutine, the imaginary roots lack physical significance for the purpose of lines plotting.

Appendix G contains an example of a data set that was first faired then splinned and then interpolated at points equal to one-twentieth of the domain of the independent variable. Once again it should be emphasized that, in order to fair a curved segment, at least six data points must be defined in that segment, including the end points of the segment.

6. Conclusions and Recommendations

6.1 Hull Form Modification

When work on this thesis began, the initial goal was to develop a series of destroyer-like hull forms for future use in seakeeping analysis. Preliminary efforts, using the method of longitudinally shifting sections [2,8], while showing promise, indicated that additional work would be required if the procedure was to apply accurately to destroyer type ships. Specifically, the method had to be adapted to ships whose maximum beam and section of maximum area did not lie at midships. These necessary changes were made successfully and the method was also extended to provide control over the ship's centerline profile. The primary motivation for this extension was to gain control over the hull form in the region of a sonar dome. While there was some apprehension about the criticality of changes to the geometry of the sonar dome, a telephone call to the Naval Sea Systems Command in Washington, D.C. [21], indicated that because of acoustic and hydrodynamic considerations the dome design should be maintained unchanged.

The resulting procedure for modifying hull forms does provide good results for that portion of the ship below the design waterline. However, as outlined in chapter two and

Appendix C, there are situations where the method does not provide exact results, e.g., when the station of maximum beam and section of maximum area do not coincide. Another unresolved weakness of the modification scheme is that it still does not provide the degree of control over specific hull regions often desired, e.g., an attempt to preserve the configuration of a sonar dome will result in preservation of the centerline profile only, the three dimensional geometry of the dome will be uncontrollably altered.

In summary it has been concluded that the modification technique holds a great deal of promise for use with automated methods. In particular, the procedure as it currently exists, will provide excellent results when dealing with ships for which there is no rigid requirement to keep a specific region fixed. Not only are the desired coefficients and characteristics obtained, the resulting hull forms appear to be acceptably fair.

As with virtually all work of this type there is still need for additional development. Specifically, it is felt that those aspects worthy of attention are:

1. Investigate a means of controlling the resulting hull form above the design waterline. At present, excessive flair or tumblehome frequently occurs.

2. Investigate a means of rigidly controlling the geometry of a specific region of the hull. This would provide a solution to the problem of keeping the sonar dome unaltered.
3. Develop a computer program to carry out the extensive mathematical and graphical calculations required by the method.

6.2 Mathematical Representation of Lines and Fairing

6.2.1 Lines Representation

In chapter three it was demonstrated, by variational calculus and by simple beam theory, that a third degree or cubic polynomial could be used to approximate the shape taken by the draftsman's spline. However, it was also pointed out that the simple cubic polynomial became indeterminate if the curves contained infinite slopes. For this reason, and also because they are capable of representing multivalued functions, the parametric cubic equations of reference [15] were incorporated into this thesis. The results obtained using this method have proven to be excellent. Not only does the technique lend itself readily to being programmed, the parametric form of the curve allows the user to define either variable as being the independent variable for the purposes of interpolation. The benefit of this capability will become apparent in the discussion of cross fairing.

The only disadvantage, as seen by this author, to using the parametric equations is that they require the user to calculate the, up to three real roots, of the polynomials each time an interpolated value is sought. However, this is not seen as being restrictive since a closed form solution exists for calculating these roots and is in fact utilized in

subroutine CALCT. Therefore, because of its great flexibility, the parametric, or rotating spline technique of chapter three, is highly recommended for use with a lines fairing scheme involving the manipulation of specific waterlines and sections.

6.2.2 Fairing

The least-squares fairing criteria, as presented in chapter four, has shown to provide an effective means of altering the position of data points in order to obtain the desired "fairing" effect. That is, if provided with an adequate tolerance interval, the cubic spline passed through the resulting points will be void of extraneous oscillations and generally pleasing to the eye. When addressing the lines of a ship in the preliminary design phase, the fact that the lines satisfy a visual inspection is likely to be sufficient. For this reason, and also the excellent results obtained by this method in reference [16], this author has concluded that this scheme would be a candidate for a complete lines fairing program for destroyer-type ships.

6.2.3 Recommendations

It is obvious that, given the capability of representing lines mathematically and also a means to fair the points on a line, the next step would be to generate a method

which would fair, in the three-dimensional sense, and display an entire ship. This author has spent a great deal of time attempting to extend the methods of reference [16] in a more general form to accommodate the peculiarities which arrise in addressing displacement-type ships. The difficulty arrose from two sources. First, an attempt was made to treat the entire ship, i.e., the bow and stern were not truncated as was the case with other methods examined. Second, in trying to treat a large variety of ships, conveniently called displacement-type, the author was confronted with the problem of attempting to describe the myriad of lines of discontinuity which one may encounter. These lines are most frequently termed control lines and may consist of the ship's profile, in an obvious sense, to the locust of points, longitudinally, where rise of floor and bilge radius meet, in a more subtle sense. Figure 6.1, for a typical bulbous bow destroyer illustrates a few of the possibilities.

If we were to ignore the fairing algorithm itself for a moment, it can be seen that if a waterline A-A is taken in figure 6.1A there must be some means of communicating the effect of control line #6 on the waterline; where the explaination of the control lines is contained in table 6.1. For this case, the effect is to create a straight line region in A-A as projected in figure 6.1B. A tentative solution to this

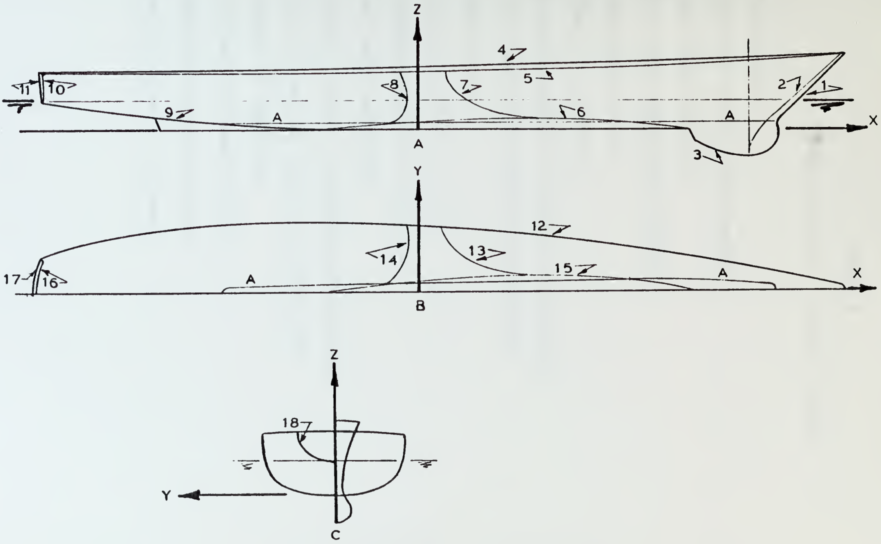


Figure 6.1 - Control Lines for a Typical Bulbous Bow Destroyer

TABLE 6.1

Explaination of Control Lines

1. Bow profile
2. Locus of stern radius centers
3. Sonar dome profile
4. Main deck centerline profile
5. Deck edge profile
6. Extent of deadrise
7. Forward extent of parallel middlebody
8. After extent of parallel middlebody
9. Keelrise aft
10. Outboard transom profile
11. Transom centerline profile
12. Deck edge waterline
13. Forward extent of parallel middlebody
14. After extent of parallel middlebody
15. Extent of deadrise
16. Outboard transom profile
17. Deckedge transom profile
18. Section view of transom

problem would be to ascribe to each control line, over a region where applicable, a code designating the effect of the control line on waterlines or sections at the point of intersection. This could be easily done by assigning another column to the CRV matrix description of the line, see section 5.1.3.

Another complication which must be resolved is: when attempting to establish the offsets for, say an arbitrary waterline, how do you seek out where this waterline intersects which control lines. In the most general case, where control lines could occur at random through a hull form this problem could prove to be formidable at least. As seen by this author, the only solution to this problem is to have only certain control lines admissible for a particular class of ship. This would necessairly limit the possible intersection combinations. The control lines shown in figure 6.1 represent, what this author feels, are typical of a contemporary destroyer.

The final aspect to be addressed is that of the cross fairing algorithm itself. Reference [16] showed that by utilizing a preassigned grid in the X-Z plane the offsets (y-coordinates) at these points could be repeatedly by faired and splined by both lines of section and waterlines. The new, or faired value of each point was taken to be the mean of that obtained by fairing the two lines. These mean values were then used as unfair data points on the lines once again

and the fairing process was repeated. This iterative procedure was continued until the movement of the points on successive iterations was less than some predefined limit. The results of this cross fairing algorithm [16] proved to be quite good. Because of this, it is felt that this procedure would also prove satisfactory for the more general method of lines fairing and representation presented in this thesis.

As a final note, this author can envision where the two independent aspects of this thesis could be combined into one program of significant value. If both the fairing procedure and lines modification techniques were automated, it would provide the designer with the capability to sketch out a rough design on the back of an envelope, specifying its fundamental coefficients and dimensions, and then by passing this information through the fairing and modification routines a faired form could be obtained. The implications of this, as a savings of time and resources, are quite astounding. If the method were further extended to permit an interactive modification of the design, an individual could literally sit down and design a faired vessel in a matter of hours instead of days.

REFERENCES

1. Loukakis, T., Chryssostomidis, C., "Seakeeping Standard Series for Cruiser-Stern Ships", Transactions, Society of Naval Architects and Marine Engineers, Vol. 83, 1975, pp. 67-127.

2. Lackenby, H., "On the Systematic Geometrical Variation of Ship Forms", Transactions, Royal Institute of Naval Architecture, Vol. 92, 1950, pp. 289-315.

3. Chryssostomidis, C., "Computer Aided Ship Design", Paper, New England Section Society of Naval Architects and Marine Engineers, May 1978.

4. Comstock, J.P., "Principles of Naval Architecture", Revised, The Society of Naval Architects and Marine Engineers, 1967, New York, New York.

5. Coons, S.A., "Surfaces for Computer-Aided Design of Space Figures", M.I.T., ESL Memorandum 9442-M-139, July 1965.

6. Gertler, M., "A Reanalysis of the Original Test Data for the Taylor Standard Series", TMB Report 806, March 1954.

7. Söding, H. and Rabein, U., "Hull Surface Design by Modifying an Existing Hull", Paper presented at the First International Symposium on Computer Aided Hull-Surface Definition, Annapolis, MD., September 1977.

8. Moor, D.I., "Effects on Performance in Still Water and Waves of Some Geometric Changes to the Form of a Large Twin-Screw Ship", Transactions, Society of Naval Architectects and Marine Engineers, Vol. 78, 1970, pp. 88-150.

9. Mehlum, E., "Variational Criteria for Smoothness", Paper for Central Institute for Industrial Research, Oslo, Norway, December, 1969.

10. Hildebrand, F.B., "Advanced Calculus for Applications", Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.

11. Higdon, A., Ohlsen, E.H., Stiles, W.B., Weese, J.A., "Mechanics of Materials", Second Edition, John Wiley and Sons, Inc., New York, July, 1968.

12. Thomas, G.B., "Calculus and Analytic Geometry", Fourth-Edition, Addison-Wesley Publishing Co., Reading, Massachusetts, June, 1972.

13. Yeung, R.W., Class Notes, MIT Ocean Engineering Department Course 13.50, Spring, 1977.

14. Carnahan, B., Wilkes, J.E., "Digital Computing and Numerical Methods", John Wiley and Sons, Inc., New York, 1973.

15. Söding, H., "Numerical Ship Lofting and Hull Form Design", Unpublished paper, Circa 1962.

16. Kyrkos, B., "The Fairing and Mathematical Representation of the Surface of a Ship Using a Small Computer", (Greek) Translations from National Technical University of Athens, Diploma Thesis, 1976.

17. Corin, T., "Recent Developments in Ship Lines Fairing at the David Taylor Model Basin", DTMB Report, Applied Mathematics Laboratory.

18. Kerwin, J., "Fitting Curves with Infinite Slopes at x=0", Unpublished paper used for Course 13.50, Spring, 1977.

19. Viega, J.P.C., "Hydrostatic Considerations in the Design of Ships with Unusual Shapes", Massachusetts Institute of Technology, Department of Ocean Engineering, Thesis, January, 1975.

20. Baumeister, T. and Marks, L.S., "Standard Handbook for Mechanical Engineers", Seventh Edition, McGraw-Hill Book Company, New York, 1967.

21. Silverstein, S., Telephone Conversation, Shipboard Sonar Group, Naval Sea Systems Command, Washington, D.C., Circa, November, 1977.

APPENDIX A

Calculation of coefficient c and centroid of the sliver of added area. Referring to figure 2.4.

Recall: $\delta x = cx(1-x)$

$$\delta\phi = \int_0^1 \delta x \, dy = c \int_0^1 x(1-x)\,dy$$

$$= c \left\{ \int_0^1 x \, dy - \int_0^1 x^2 dy \right\} = c[\phi - 2\phi\overline{x}]$$

$$c = \frac{\delta\phi}{\phi - 2\phi\overline{x}} = \frac{\delta\phi}{\phi(1-2\overline{x})}$$

$$\delta x = \frac{\delta\phi}{\phi(1-2\overline{x})} x(1-x) \qquad (A.1)$$

solving for centroid, h

$$\delta\phi \cdot h = \int_0^1 \delta x \left(x + \frac{\delta x}{2}\right) dy$$

$$= \int_0^1 x\delta x \, dy + \frac{1}{2} \int_0^1 \delta x^2 dy$$

substituting for δx

$$\delta\phi \cdot h = \frac{\delta\phi}{\phi(1-2\overline{x})} \int_0^1 (x^2 - x^3)\,dy + \frac{\delta x^2}{2(1-2\overline{x})^2} \int_0^1 (x^2 - 2x^3 + x^4)\,dy$$

$$h = \frac{2\overline{x} - 3k^2}{1 - 2\overline{x}} + \frac{\delta\phi}{\phi} \left\{ \frac{\overline{x} - 3k^2 + 2r^3}{(1 - 2\overline{x})^2} \right\} \qquad (A.2)$$

For $\delta\phi << \phi$

$$h \sim \frac{2\bar{x} - 3k^2}{1-2\bar{x}} \tag{A.3}$$

where:

$$k^2 = \frac{1}{3\phi} \int_0^1 x^3 dy \qquad r^3 = \frac{1}{4\phi} \int_0^1 x^4 dy$$

APPENDIX B

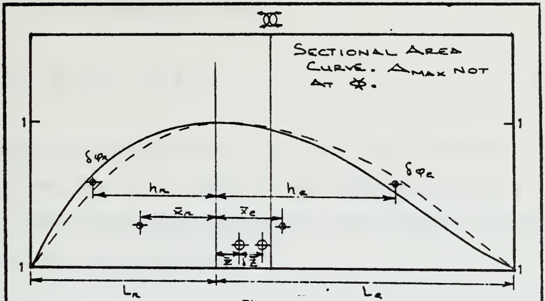


Figure B.1

Referring to figure B.1 above, all longitudinal dimensions are measured with respect to the point of maximum sectional area. Assume:

$$\delta x = cx(1-x)$$

$$= \frac{\delta\phi}{\phi(1-2\bar{x})}\, x(1-x)$$

For the new hull form

$$\phi_t' = \frac{1}{L}\ \{L_e(\phi_e + \delta\phi_e) + L_r(\phi_r + \delta\phi_r) \qquad (B.1)$$

$$\bar{z}' = \frac{1}{L^2\phi_t'}\{L_e^2(\phi_e\bar{x}_e + \delta\phi_e h_e) - L_r^2(\phi_r\bar{x}_r + \delta\phi_r h_r) \qquad (B.2)$$

Also for the original hull form

$$\phi_t = \frac{1}{L}\{L_e\phi_e + L_r\phi_r\} \qquad (B.3)$$

We now have to apply equations (B.1) and (B.2) to obtain values of $\delta\phi_e$ and $\delta\phi_r$ in terms of the known quantities $\bar{z}$ and ϕ_t'. These quantities representing the desired values LCB and C_p for the derived form.

Solving equation (B.1) for $\delta\phi_e$.

$$\phi_t'L = L_e\phi_e + L_e\delta\phi_e + L_r(\phi_r + \delta\phi_r)$$

$$\delta\phi_e = \frac{1}{L_e}\{L\phi_t' - L_e\phi_e - L_r(\phi_r + \delta\phi_r)\} \qquad (B.4)$$

Substituting equation (B.4) into equation (B.2) and solving for $\delta\phi_r$.

$$\bar{z}'L^2\phi_t' = L_e^2(\phi_e\bar{x}_e + \delta\phi_e h_e) - L_r^2(\phi_r\bar{x}_r + \delta\phi_r h_r)$$

expanding the right hand side, R.H.S.

$$\text{L.H.S.} = L_e^2\phi_e\bar{x}_e + L_e^2\delta\phi_e h_e - L_r^2\phi_r\bar{x}_r - L_r^2\delta\phi_r h_r$$

rearranging terms

$$L_r^2\delta\phi_r h_r - L_e^2\delta\phi_e h_e = L_e^2\phi_e\bar{x}_e - L_r^2\phi_r\bar{x}_r - \bar{z}'L^2\phi_t'$$

substituting in L.H.S.

$$L_r^2\delta\phi_r h_r - L_e^2 h_e \frac{1}{L_e}\{L\phi_t' - L_e\phi_e - L_r(\phi_r + \delta\phi_r)\} = \text{R.H.S.}$$

$$L_r^2\delta\phi_r h_r - L_e h_e L\phi_t' + L_e^2 h_e\phi_e + L_r L_e h_e\phi_r + L_r L_e h_e\delta\phi_r =$$

$$\delta\phi_r(L_r^2 h_r + L_r L_e h_e) + L_e(L_e h_e\phi_e + L_r h_e\phi_r - h_e L\phi_t') =$$

$$\delta\phi_r = \frac{1}{L_r^2 h_r + L_r L_e h_e}\{L_e^2\phi_e\bar{x}_e + L_r^2\phi_r\bar{x}_r - \bar{z}'L^2\phi_t' - L_e h_e(L_e\phi_e + L_r\phi_r - L\phi_t')\} \qquad (B.5)$$

Equation (B.5) may be substituted back into equation (B.4) to obtain a value for $\delta\phi_e$.

However, if the simplified form for h is not used

$$h = f(\delta\phi_r, \delta\phi_e)$$

Derivation of $\delta\phi_e$ from equation (1)

$$\delta\phi_r = \frac{1}{L_r}\{L\phi'_t - L_e\phi_e - L_e\delta\phi_e - L_r\phi_r\} \tag{B.6}$$

substituting into equation (B.2) and rearranging

$$\bar{z}'L^2\phi'_t - L_e^2\phi_e\bar{x}_e + L_r^2\phi_r\bar{x}_r = \text{R.H.S.}$$

$$= L_e^2h_e\delta\phi_e - L_rh_r\{L\phi'_t - L_e\phi_e - L_e\delta\phi_e - L_r\phi_r\}$$

$$= \delta\phi_e\{L_e^2h_e + L_rL_eh_r\} - L_rh_r\{L\phi'_t - L_e\phi_e - L_r\phi_r\}$$

$$\delta\phi_e = \frac{1}{L_e^2h_e + L_rL_eh_r}\{L_r^2\phi_r\bar{x}_r - L_e\phi_e\bar{x}_e + \bar{z}'L^2\phi'_t - L_rh_r(L_e\phi_e + L_r\phi_r - L\phi'_t)\} \tag{B.7}$$

The form of this equation is merely the transposition of subscripts by r→e→r of equation (B.5) for $\delta\phi_r$.

APPENDIX C

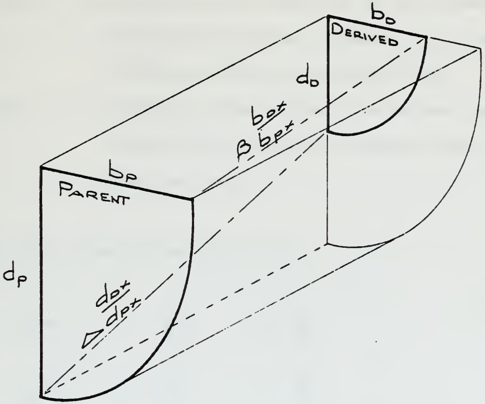


Figure C.1 - Section Modification

Definitions:

a = area of section

b = beam of section

d = draft of section

Subscripts:

p = parent hull form

d = derived hull form

x = a maximum value, e.g., b_{dx} is the maximum value of the beam in the derived hull form

Superscript:

bar, (—): refers to the sections in the parent and derived form which interact to create the derived section of maximum area.

star, (*): refers to the sections which are used to create the section of maximum beam in the derived form.

Further define the following ratios:

$$A = \frac{a}{a_x}, \qquad B = \frac{b}{b_x}, \qquad D = \frac{d}{d_x}$$

$$\alpha = \frac{A_d}{A_p}, \qquad \beta = \frac{B_d}{B_p}, \qquad \Delta = \frac{D_d}{D_p}$$

also

$$\frac{A}{B \cdot D} = R$$

It is by selecting a section in the parent, whose value of R is equal to that of the derived section being sought, that the new sections are created. It should also be evident by referring to figure C.1, that the area of the derived section will be as follows:

$$a_D = a_p \, \beta \, \frac{b_{dx}}{b_{px}} \, \Delta \, \frac{d_{dx}}{d_{px}}$$

It remains to be shown that in some cases the resulting area ratio is not always what is desired.

The following expression will also be useful.

$$a_{dx} = \bar{a}_p \bar{\beta} \, \frac{b_{dx}}{b_{px}} \, \bar{\Delta} \, \frac{d_{dx}}{d_{px}}$$

For any derived section, the area ratio obtained is:

$$\frac{a_d}{a_{dx}} = a_p \beta \, \frac{b_{dx}}{b_{px}} \, \Delta \, \frac{d_{dx}}{d_{px}} \left\{ \frac{1}{\bar{a}_p \bar{\beta} \, \frac{b_{dx}}{b_{px}} \, \bar{\Delta} \, \frac{d_{dx}}{d_{px}}} \right\} \qquad \text{(C.1)}$$

$$= \frac{a_p}{\bar{a}_p} \, \frac{\beta}{\bar{\beta}} \, \frac{\Delta}{\bar{\Delta}}$$

However, the area ratio desired is:

$$R_p B_d D_d = \frac{\frac{a_p}{a_{px}}}{\frac{b_p}{b_{px}} \cdot \frac{d_p}{d_{px}}} \left\{ \frac{b_d}{b_{dx}} \, \frac{d_d}{d_{dx}} \right\} = \frac{a_p}{a_{px}} \, \beta \, \Delta \qquad \text{(C.2)}$$

Therefore, the ratio of A_d desired to A_d obtained is:

$$\frac{\frac{a_p}{a_{px}} \, \beta \, \Delta}{\frac{a_p}{\bar{a}_p} \, \frac{\beta}{\bar{\beta}} \, \frac{\Delta}{\bar{\Delta}}} = \frac{\bar{a}_p}{\bar{a}_{px}} \, \bar{\beta} \, \bar{\Delta} = \frac{\bar{a}_p}{a_{px}} \, \frac{\bar{B}_d}{\bar{B}_p} \, \frac{\bar{D}_d}{\bar{D}_p}$$

$$= \bar{R}_p \bar{B}_d \bar{D}_d \qquad \text{(C.3)}$$

Define: $$S = \frac{1}{\bar{R}_p \bar{B}_p \bar{D}_d}$$

It can be readily seen that, if the sectional area curve and the design waterline have their maximum values at the same longitudinal position, the value of S will be 1, i.e., $S = 1$. Hence, the area curve obtained will be equal to that desired.

If this is not the case, the designer has one option which will permit him to create a ship with the desired values of C_p, LCB, C_w and LCF.

- The designer must select a common longitudinal position about which to alter both curves. This will permit him to freeze either the point of maximum section of the point of maximum beam. Not both.
- The only other alternative is to carry out the original procedure and accept slightly different values of C_p and C_w. LCB and LCF will be as desired. The factor by which C_w will differ is:

$$W = \frac{R_p^* D_d^*}{A_d^*} \tag{C.5}$$

APPENDIX D, [15]

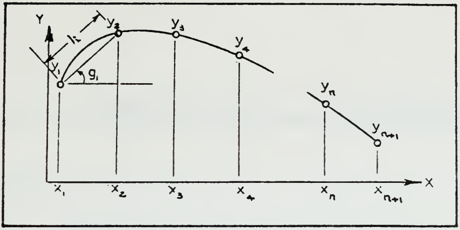


Figure D.1 - Rotating Spline Routine

For n intervals bounded by n+1 points the curve between points P_i and P_{i+1} may be computed as follows:

1. Compute initially:

$$a.\quad g_1 = \begin{cases} \arctan\,[(y_2-y_1)/(x_2-x_1)], & \text{if } x_1 \neq x_2 \\ \pi/2, & \text{if } x_1 = x_2 \end{cases}$$

$$b.\quad p_1 = g_1$$

2. Compute n times for $i = 1(1)n$

a. $\ell_i = \{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2\}^{1/2}$

b.

$$g_i = g_{i-1} + \arctan\left\{\frac{(y_{i+1}-y_i)(x_i-x_{i-1})-(y_i-y_{i-1})(x_{i+1}-x_i)}{(x_{i+1}-x_i)(x_i-x_{i-1})+(y_{i+1}-y_i)(y_i-y_{i-1})}\right\}$$

only if $i > 1$

c. $k_i = \begin{cases} -1/2, \text{ if } i = 1 \\ \dfrac{-1}{2+(k_{i-1}+2)\,\ell_i/\ell_{i-1}}, & \text{if } i > 1 \end{cases}$

d. $r_i = \dfrac{3k_i\ (p_i-g_i)}{k_i+2}$

e. $p_{i+1} = \begin{cases} p_i-r_i(1+1/k_i), \text{ if } k_i \neq 0 \\ \dfrac{3g_i-p_i}{2}, \text{ if } k = 0 \end{cases}$

3. Compute once:

a. $q_{n+1} = 0$

b. $d_{n+1} = p_{n+1} + q_{n+1}$

4. Compute n times for i = 1(-1)n

a. $q_i = r_i + q_{i+1} k_i$

b. $d_i = p_i + q_i$

c. $a_i = \tan(d_i - g_i)$

d. $b_i = \tan(d_{i+1} - g_i)$

The above procedure applies to the case where the ends are pinned, i.e., $d^2y/dx^2 = 0$ at $x = x_1$ and $x = x_{n+1}$. If, however, it is desired to have the beginning slope equal to t_1, the following changes must be made:

eqn. 1.b. $p_1 = t_1$

2.c. $k_1 = 0$

If it is desired to specify the end slope as t_{n+1}:

eqn. 3.a. $q_{n+1} = t_{n+1} - p_{n+1}$

To interpolate any point on the curve the following parametric equations are used:

$$x = x_i+(x_{i+1}-x_i)t-(y_{i+1}-y_i)t(1-t)[a_i(1-t)-b_it]$$

$$y = y_i+(y_{i+1}-y_i)t-(x_{i+1}-x_i)t(1-t)[a_i(1-t)-b_it]$$

$$\text{for } 0 \leq t \leq t$$

Section 3.2.4 describes the actual interpolation procedure.

APPENDIX E

The following procedure was taken from reference [20] and is that used to obtain the roots of the parametric equations for x and y shown in Appendix D.

Given the general form of the cubic polynomial:

$$x^3 + ax^2 + bx + c = 0 \tag{E.1}$$

this may be reduced to the following by dividing by $x = x_1 - a/3$:

$$x_1^3 = Ax_1 + B \tag{E.2}$$

where

$$A = 3(a/3)^2 - b$$

$$B = -2(a/3)^3 + b(a/3) - c \tag{E.3}$$

Defining

$$p = A/3 \text{ and } q = B/2 \tag{E.4}$$

The roots of equation (E.2) are as follows:

Case I: $q^2 - p^3 > 0$, there is one real root

$$x_1 = \{q + \sqrt{q^2-p^3}\}^{1/3} + \{q - \sqrt{q^2-p^3}\}^{1/3} \qquad (E.5)$$

There are also two complex conjugate roots.

Case II: $q^2 - p^3 = 0$, there are three real roots of which two are repeated, i.e., only two roots are unique.

$$x_1 = 2(q)^{1/3};\ x_2 = -(q)^{1/3};\ x_3 = x_2 \qquad (E.6)$$

Case III: $q^2 - p^3 < 0$, there are three real and distinct roots.

$$x_1 = 2\sqrt{p}\cos(U/3)$$

$$x_2 = 2\sqrt{p}\cos(U/3 + 2\pi/3) \qquad (E.7)$$

$$x_3 = 2\sqrt{p}\cos(U/3 + 4\pi/3)$$

where

$$\cos U = q/p\sqrt{p}$$

$$0 \leq U \leq \pi \qquad (E.8)$$

NOTE: These are roots of equation (E.2). To obtain the roots of equation (E.1) $-p/3$ must be added to the above solutions.

APPENDIX F

Figures F.1 and F.2 are conceptual flowcharts of the fairing and the splinning and interpolation procedures respectively. In the program listing that follows there is a short MAIN segment that requests the input data for a specific line and generates twenty-one (including end points) interpolated data points. While this program segment might prove of some value, it was designed primarily to test the various subroutines.

NOTE: The program as listed requires the use of the LEQT1F Subroutine from the IMSL library. This subroutine is used in STRIP1 and STRIP2 to solve a 4x4 and a 3x3 system of simultaneous linear equations. For these SMAT is the coefficient matrix and T is the resultant column vector. If this library is not available any equivalent procedure could be substituted.

NOTE: Due to time constraints at the time of publication, the program, as listed, will not accommodate curves with point types three or five. It will, however, handle curves without straight line segments and infinite slopes at end points.

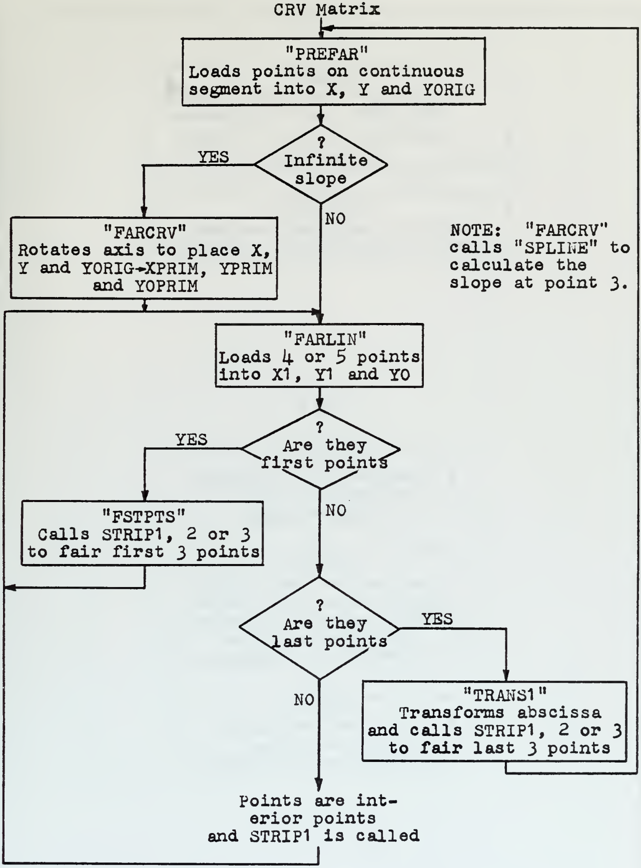

CRV Matrix
"PREFAR"
Loads points on continuous segment into X, Y and YORIG
?
Infinite slope
YES
NO
"FARCRV"
Rotates axis to place X, Y and YORIG→XPRIM, YPRIM and YOPRIM
NOTE: "FARCRV" calls "SPLINE" to calculate the slope at point 3.
"FARLIN"
Loads 4 or 5 points into X1, Y1 and YO
?
Are they first points
YES
NO
"FSTPTS"
Calls STRIP1, 2 or 3 to fair first 3 points
?
Are they last points
YES
NO
"TRANS1"
Transforms abscissa and calls STRIP1, 2 or 3 to fair last 3 points
Points are interior points and STRIP1 is called

Figure F.1 — Flowchart of Fairing Subroutines

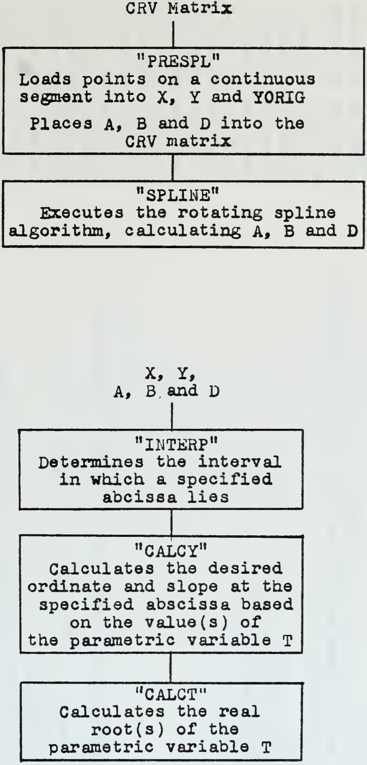

CRV Matrix
"PRESPL"
Loads points on a continuous segment into X, Y and YORIG
Places A, B and D into the CRV matrix
"SPLINE"
Executes the rotating spline algorithm, calculating A, B and D
X, Y,
A, B and D
"INTERP"
Determines the interval in which a specified abcissa lies
"CALCY"
Calculates the desired ordinate and slope at the specified abscissa based on the value(s) of the parametric variable T
"CALCT"
Calculates the real root(s) of the parametric variable T

Figure F.2 —— Flowchart of Splinning and Interpolation Subroutines

```
      COMMON /COM1/SMAT(4,4),T(4),WKAREA(4)
      COMMON /COM2/X(30),Y(30),YORIG(30)
      COMMON /COM3/X1(5),Y1(5),YO(5)
      COMMON /COM4/IPAIR,ACC,LIMIT
      COMMON /COM5/XPRIM(30),YPRIM(30),YOPRIM(30)
      COMMON /COM6/XCRV(30),YCRV(30),YOCRV(30)
      COMMON /COM7/A(30),B(30),D(30)
      COMMON /COM8/Q
      COMMON /COM9/CRV(32,7)
      COMMON /COM10/G(30),S(30),C(30),QS(30),R(30),P(30)
      COMMON /COM11/YINT(10),DYDX(10)
      COMMON /COM12/IPT(30)
      COMMON /COM13/KCRV
      COMMON /COM14/RT(3),KFLG2
      WRITE (6,1000)
 1000 FORMAT(' ','PREPARE TO ENTER LINES 31 & 32 OF CRV ARRAY.')
C
C
C
      DO 1 M=1,4
      DO 2 MM=1,4
 2    SMAT(M,MM)=0.
 1    CONTINUE
      READ (5,*) (CRV(31,I), I=1,4)
      READ (5,*) (CRV(32,I), I=1,3)
      NPTS=INT(CRV(31,2)+.1)
      DO 10 I=1,NPTS
      IF (I.EQ.1) WRITE(6,99) NPTS
 99   FORMAT('0','BEGIN ENTERING DATA FOR THE ',I2,' DATA POINTS.')
      READ (5,*) CRV(I,1),CRV(I,2),CRV(I,4)
 10   CRV(I,3)=CRV(I,2)
      WRITE (6,101)
 101  FORMAT(' ','ENTER ACC, TOL AND LIMIT.')
      READ (5,*) ACC,TOL,LIMIT
      CALL PREPAR(TOL)
      DO 102 I=1,NPTS
```

```
      IF (I.EQ.1) WRITE (6,104)                                         00000370 MAIN0037
  104 FORMAT(' ',' ORIGINAL ',10X,'  PAIRED  ')                         00000380 MAIN0038
      WRITE (6,103) CRV(I,2),CRV(I,3)                                   00000390 MAIN0039
  103 FORMAT(' ',F10.4,10X,F10.4)                                       00000400 MAIN0040
  102 CONTINUE                                                          00000410 MAIN0041
      WRITE (6,301)                                                     00000420 MAIN0042
  301 FORMAT ('0','PRESPL AND SPLINE CHECK')                            00000430 MAIN0043
      DO 302 I=1,NPTS                                                   00000440 MAIN0044
      X(I)=CRV(I,1)                                                     00000450 MAIN0045
      Y(I)=CRV(I,3)                                                     00000460 MAIN0046
      IPT(I)=INT(CRV(I,4)+.1)                                           00000470 MAIN0047
  302 YORIG(I)=CRV(I,2)                                                 00000480 MAIN0048
      CALL PRESPL                                                       00000490 MAIN0049
      WRITE (6,201)                                                     00000500 MAIN0050
  201 FORMAT(' ','INTERPOLATION CHECK')                                 00000510 MAIN0051
      NP1=NPTS+1                                                        00000520 MAIN0052
      WRITE (6,303)                                                     00000530 MAIN0053
  303 FORMAT ('0','   XINT    ',10X,'    YINT    ',10X,'    DYDX')      00000540 MAIN0054
      DX=(X(NPTS)-X(1))/20.                                             00000550 MAIN0055
      DO 105 I=1,21                                                     00000560 MAIN0056
      XINT=X(1)+FLOAT(I-1)*DX                                           00000570 MAIN0057
      CALL INTERP(NP1,XINT,KFLG1)                                       00000580 MAIN0058
      IF (KFLG1.EQ.0) GO TO 105                                         00000590 MAIN0059
      DO 106 J=1,KFLG1                                                  00000600 MAIN0060
      WRITE (6,107) XINT,YINT(J),DYDX(J)                                00000610 MAIN0061
  107 FORMAT(' ',F10.4,10X,F10.4,10X,F10.4)                             00000620 MAIN0062
  106 CONTINUE                                                          00000630 MAIN0063
  105 CONTINUE                                                          00000640 MAIN0064
      STOP                                                              00000650 MAIN0065
      END                                                               00000660 MAIN0066
```

```
C  THIS SUBROUTINE TAKES CURVE DATA IN A 2-D ARRAY AND SPLINES IT TO     00000670 PRSP0001
C   DETERMINE COEFFICIENTS A,B AND ANGLES D.  IE. CRV(XX,5 6 OR7).       00000680 PRSP0002
      SUBROUTINE PRESPL                                                   00000690 PRSP0003
      COMMON /COM2/X(30),Y(30),YORIG(30)                                  00000700 PRSP0004
      COMMON /COM7/A(30),B(30),D(30)                                      00000710 PRSP0005
      COMMON /COM9/CRV(32,7)                                              00000720 PRSP0006
      NPTS=INT(CRV(31,2)+.1)                                              00000730 PRSP0007
      MARK=0                                                              00000740 PRSP0008
      DO 101 I=1,NPTS                                                     00000750 PRSP0009
      IF (I.EQ.1.OR.INT(CRV(I,4)+.1).EQ.0.OR.INT(CRV(I,4)+.1).EQ.1) GO T00000760 PRSP0010
     1O 201                                                               00000770 PRSP0011
      IF (INT(CRV(I,4)+.1).EQ.3.OR.INT(CRV(I,4)+.1).EQ.5) GO TO 401       00000780 PRSP0012
  102 KP1=K+1                                                             00000790 PRSP0013
      IF (K.EQ.I) KP1=I                                                   00000800 PRSP0014
      X(KP1)=CRV(I,1)                                                     00000810 PRSP0015
      Y(KP1)=CRV(I,3)                                                     00000820 PRSP0016
      K=KP1                                                               00000830 PRSP0017
      GO TO 401                                                           00000840 PRSP0018
  101 CONTINUE                                                            00000850 PRSP0019
  999 RETURN                                                              00000860 PRSP0020
  601 EC=EC1+EC2                                                          00000870 PRSP0021
      CALL SPLINE(K,EC,ES1P,ES2P)                                         00000880 PRSP0022
      IF (I.EQ.NPTS) GO TO 702                                            00000890 PRSP0023
      DO 701 L=1,K                                                        00000900 PRSP0024
      LC=I-L                                                              00000910 PRSP0025
      LK=K-L+1                                                            00000920 PRSP0026
      CRV(LC,5)=A(LK)                                                     00000930 PRSP0027
      CRV(LC,6)=B(LK)                                                     00000940 PRSP0028
  701 CRV(LC,7)=D(LK)                                                     00000950 PRSP0029
      GO TO 101                                                           00000960 PRSP0030
  702 DO 703 L=1,K                                                        00000970 PRSP0031
      LC=I+1-L                                                            00000980 PRSP0032
      LK=K+1-L                                                            00000990 PRSP0033
      CRV(LC,5)=A(LK)                                                     00001000 PRSP0034
      CRV(LC,6)=B(LK)                                                     00001010 PRSP0035
  703 CRV(LC,7)=D(LK)                                                     00001020 PRSP0036
```

```
      GO TO 101                                                         00001030 PRSP0037
C                                                                       00001040 PRSP0038
C                                                                       00001050 PRSP0039
  201 K=I-MARK                                                          00001060 PRSP0040
      IF (K.EQ.2.AND.MARK.NE.0) GO TO 501                               00001070 PRSP0041
  202 X(K)=CRV(I,1)                                                     00001080 PRSP0042
      Y(K)=CRV(I,3)                                                     00001090 PRSP0043
      IF (I.EQ.NPTS) GO TO 102                                          00001100 PRSP0044
      GO TO 101                                                         00001110 PRSP0045
  501 IM1=I-1                                                           00001120 PRSP0046
      X(1)=CRV(IM1,1)                                                   00001130 PRSP0047
      Y(1)=CRV(IM1,3)                                                   00001140 PRSP0048
      GO TO 202                                                         00001150 PRSP0049
  401 MARK=I-1                                                          00001160 PRSP0050
C  CALCULATE BEGINNING END CONDITION.                                   00001170 PRSP0051
      KM1=K-1                                                           00001180 PRSP0052
      IF (I-K.EQ.1.OR.I-K.EQ.0) GO TO 301                               00001190 PRSP0053
      IMK=I-K                                                           00001200 PRSP0054
      IF (INT(CRV(IMK,4)+.1).EQ.0.OR.INT(CRV(IMK,4)+.1).EQ.1) GO TO 302 00001210 PRSP0055
      GO TO 303                                                         00001220 PRSP0056
C  CALCULATE END CONDITION.                                             00001230 PRSP0057
  402 IF (I.EQ.NPTS) GO TO 304                                          00001240 PRSP0058
      IF (INT(CRV(I,4)+.1).EQ.3.OR.INT(CRV(I,4)+.1).EQ.5) GO TO 305     00001250 PRSP0059
      IF (INT(CRV(I,4)+.1).EQ.0.OR.INT(CRV(I,4)+.1).EQ.1) GO TO 306     00001260 PRSP0060
      GO TO 999                                                         00001270 PRSP0061
C                                                                       00001280 PRSP0062
  301 ES1P=CRV(31,3)                                                    00001290 PRSP0063
      EC1=FLOAT(INT(CRV(32,2)))                                         00001300 PRSP0064
      GO TO 402                                                         00001310 PRSP0065
C                                                                       00001320 PRSP0066
  302 ES1P=CRV(IMK,7)                                                   00001330 PRSP0067
      EC1=2.                                                            00001340 PRSP0068
      IF (INT(CRV(IMK,4)+.1).EQ.0) EC1=3.                               00001350 PRSP0069
      GO TO 402                                                         00001360 PRSP0070
C                                                                       00001370 PRSP0071
  303 IMKM1=IMK-1                                                       00001380 PRSP0072
```

```
      ES1P=ATAN((Y(IMK)-Y(IMKM1))/(X(IMK)-X(IMKM1)))
      EC1=3.
      GO TO 402
C
  304 ES2P=CRV(32,3)
      EC2=CRV(32,2)-FLOAT(INT(CRV(32,2)))
      GO TO 601
C
  305 IM1=I-1
      ES2P=ATAN((CRV(I,3)-CRV(IM1,3))/(CRV(I,1)-CRV(IM1,1)))
      EC2=3.
      GO TO 601
C
  306 EC2=2.
      GO TO 601
      END
```

```
C THIS SUBROUTINE CALCULATES THE DATA NECESSARY TO GENERATE A CUBIC
C  PCLYNOMIAL THROUGH A SERIES OF GIVEN POINTS.  THE ALGORITHM USED WAS
C  DEVELOPED BY PROFESSOR H. SODING AND INVOLVES THE INCREMENTAL
C  ROTATION OF THE AXES TO ACCOMODATE INFINITE SLOPES.
      SUBROUTINE SPLINE(NP1,EC,ES1,ES2)
      COMMON /COM2/X(30),Y(30),YORIG(30)
      COMMON /COM7/A(30),B(30),D(30)
      COMMON /COM10/G(30),S(30),C(30),QS(30),R(30),P(30)
      INTEGER DEC
      IND=INT(EC)
      DEC=INT((EC-FLOAT(IND)+.01)*10.)
      PI=ACOS(-1.)
      G(1)=PI/2.
      IF (X(2).NE.X(1)) G(1)=ATAN((Y(2)-Y(1))/(X(2)-X(1)))
      P(1)=G(1)
      N=NP1-1
      IF (IND.EQ.3) P(1)=ES1
      IF (IND.EQ.4.AND.Y(1).GT.Y(2)) P(1)=-PI/2.
      IF (IND.EQ.4.AND.Y(1).LE.Y(2)) P(1)=PI/2.
      DO 101 I=1,N
      IP1=I+1
      IM1=I-1
      S(I)=SQRT((X(IP1)-X(I))**2+(Y(IP1)-Y(I))**2)
      A2=X(I)-X(IM1)
      A3=Y(I)-Y(IM1)
      A1=Y(IP1)-Y(I)
      A4=X(IP1)-X(I)
      IF (I.GT.1) G(I)=G(IM1)+ATAN((A1*A2-A3*A4)/(A4*A2+A1*A3))
      C(I)=-.5
      IF (IND.EQ.3.OR.IND.EQ.4) C(1)=0.
      IF (I.GT.1) C(I)=-1./(2.+(C(IM1)+2.)*S(I)/S(IM1))
      R(I)=3.*C(I)*(P(I)-G(I))/(C(I)+2.)
      P(IP1)=(3.*G(I)-P(I))/2.
  101 IF (C(I).NE.0.) P(IP1)=P(I)-R(I)*(1.+1./C(I))
      QS(NP1)=0.
      IF (DEC.EQ.3.OR.DEC.EQ.4)  QS(NP1)=ES2-P(NP1)
```

```
      D(NP1)=P(NP1)+QS(NP1)                                             00001910 SPLN0037
      DO 102 I=1,N                                                      00001920 SPLN0038
      K=N+1-I                                                           00001930 SPLN0039
      KP1=K+1                                                           00001940 SPLN0040
      QS(K)=R(K)+QS(KP1)*C(K)                                           00001950 SPLN0041
      D(K)=P(K)+QS(K)                                                   00001960 SPLN0042
      A(K)=TAN(D(K)-G(K))                                               00001970 SPLN0043
  102 B(K)=TAN(D(KP1)-G(K))                                             00001980 SPLN0044
      RETURN                                                            00001990 SPLN0045
      END                                                               00002000 SPLN0046
```

```
C  THIS SUBROUTINE CALCULATES INTERPOLATED VALUES OF POSITION AND SLOPE 00002010 INTP0001
C   OF A CURVE THAT HAS BEEN SPLINED PREVIOUSLY.  IT WILL GIVE MULTIPLE 00002020 INTP0002
C   VALUES IF THE CURVE IS NOT SINGLE VALUED.                           00002030 INTP0003
      SUBROUTINE INTERP(NP1,XINT,KFLG1)                                 00002040 INTP0004
      COMMON /COM2/X(30),Y(30),YORIG(30)                                00002050 INTP0005
      COMMON /COM7/A(30),B(30),D(30)                                    00002060 INTP0006
      COMMON /COM9/CRV(32,7)                                            00002070 INTP0007
      COMMON /COM11/YINT(10),DYDX(10)                                   00002080 INTP0008
      COMMON /COM12/IPT(30)                                             00002090 INTP0009
      N=NP1-1                                                           00002100 INTP0010
      NM1=N-1                                                           00002110 INTP0011
      KFLG1=0                                                           00002120 INTP0012
      DO 99 I=1,N                                                       00002130 INTP0013
   99 IPT(I)=INT(CRV(I,4)+.1)                                           00002140 INTP0014
      DO 101 I=1,NM1                                                    00002150 INTP0015
      IP1=I+1                                                           00002160 INTP0016
      IF (XINT.EQ.X(I).AND.I.EQ.1) GO TO 201                            00002170 INTP0017
      IF (XINT.EQ.X(IP1)) GO TO 202                                     00002180 INTP0018
      IF (X(IP1).GT.X(I).AND.XINT.GT.X(I).AND.XINT.LT.X(IP1)) CALL CALCY00002190 INTP0019
     1(KFLG1,X(I),X(IP1),Y(I),Y(IP1),A(I),B(I),XINT,IPT(IP1))           00002200 INTP0020
      IF (X(IP1).LT.X(I).AND.XINT.LT.X(I).AND.XINT.GT.X(IP1)) CALL CALCY00002210 INTP0021
     1(KFLG1,X(I),X(IP1),Y(I),Y(IP1),A(I),B(I),XINT,IPT(IP1))           00002220 INTP0022
  101 CONTINUE                                                          00002230 INTP0023
      RETURN                                                            00002240 INTP0024
C                                                                       00002250 INTP0025
  201 KFLG1=KFLG1+1                                                     00002260 INTP0026
      YINT(KFLG1)=Y(I)                                                  00002270 INTP0027
      IF (D(I).NE.ACOS(-1.)/2.) DYDX(KFLG1)=TAN(D(I))                   00002280 INTP0028
      IF (D(I).EQ.ACOS(-1.)/2.) DYDX(KFLG1)=1.E50                       00002290 INTP0029
      GO TO 101                                                         00002300 INTP0030
  202 KFLG1=KFLG1+1                                                     00002310 INTP0031
      YINT(KFLG1)=Y(IP1)                                                00002320 INTP0032
      IF (D(IP1).NE.ACOS(-1.)/2.) DYDX(KFLG1)=TAN(D(IP1))               00002330 INTP0033
      IF (D(IP1).EQ.ACOS(-1.)/2.) DYDX(KFLG1)=1.E50                     00002340 INTP0034
      GO TO 101                                                         00002350 INTP0035
      END                                                               00002360 INTP0036
```

```
      SUBROUTINE CALCY(KFLG1,XI,XIP1,YI,YIP1,AI,BI,XINT,IPTIP1)         00002370 CLCY0001
      COMMON /COM11/YINT(10),DYDX(10)                                   00002380 CLCY0002
      COMMON /COM14/RT(3),KFLG2                                         00002390 CLCY0003
      IP1=I+1                                                           00002400 CLCY0004
      IF (IPTIP1.EQ.3..OR.IPTIP1.EQ.5.) GO TO 101                       00002410 CLCY0005
      CALL CALCT(XI,XIP1,YI,YIP1,AI,BI,XINT)                            00002420 CLCY0006
      IF (KFLG2.EQ.0) GO TO 301                                         00002430 CLCY0007
      A1=YIP1-YI                                                        00002440 CLCY0008
      A2=XIP1-XI                                                        00002450 CLCY0009
      DO 102 K=1,KFLG2                                                  00002460 CLCY0010
      A3=RT(K)*(1.-RT(K))                                               00002470 CLCY0011
      A4=1.-2.*RT(K)                                                    00002480 CLCY0012
      A5=1.-RT(K)                                                       00002490 CLCY0013
      A6=AI*A5-BI*RT(K)                                                 00002500 CLCY0014
      KFLG1=KFLG1+1                                                     00002510 CLCY0015
      YINT(KFLG1)=YI+A1*RT(K)+A2*A3*A6                                  00002520 CLCY0016
      A7=AI+BI                                                          00002530 CLCY0017
      A8=4.*AI+2.*BI                                                    00002540 CLCY0018
      A9=3.*RT(K)**2                                                    00002550 CLCY0019
      A10=A9*A7-RT(K)*A8+AI                                             00002560 CLCY0020
      DYDT=A1+A2*A10                                                    00002570 CLCY0021
      DXDT=A2-A1*A10                                                    00002580 CLCY0022
      DYDX(KFLG1)=1.E50                                                 00002590 CLCY0023
  102 IF (DXDT.NE.0.) DYDX(KFLG1)=DYDT/DXDT                             00002600 CLCY0024
  999 RETURN                                                            00002610 CLCY0025
C                                                                       00002620 CLCY0026
  101 KFLG1=KFLG1+1                                                     00002630 CLCY0027
      DYDX(KFLG1)=(YIP1-YI)/(XIP1-XI)                                   00002640 CLCY0028
      YINT(KFLG1)=YI+DYDX(KFLG1)*(XINT-XI)                              00002650 CLCY0029
      GO TO 999                                                         00002660 CLCY0030
  301 WRITE (6,302)                                                     00002670 CLCY0031
  302 FORMAT(' ','THERE IS NO VALUE OF T WITHIN THE LIMITS.')           00002680 CLCY0032
      GO TO 999                                                         00002690 CLCY0033
      END                                                               00002700 CLCY0034
```

```
C  THIS FUNCTION CALCULATES THE VALUE OF T USING THE CLOSED FORM CUBE    00002710 CLCT0001
C   ROOT EQUATION FOUND IN THE CRC MATH TABLES.  THE VALUE OF T SELECTED00002720 CLCT0002
C   IS THE ROOT THAT IS BOTH REAL AND LYING BETWEEN O AND 1.             00002730 CLCT0003
      SUBROUTINE CALCT(XI,XIP1,YI,YIP1,AI,BI,XINT)                      00002740 CLCT0004
      COMMON /COM14/RT(3),KFLG2                                         00002750 CLCT0005
  601 FORMAT(' ','THERE ARE MORE THAN 2 REAL ROOTS OF T.   YOU NEED MORE00002760 CLCT0006
     1DEFINING POINTS IN THIS REGION.')                                 00002770 CLCT0007
      KFLG2=0                                                           00002780 CLCT0008
      A1=XIP1-XI                                                        00002790 CLCT0009
      A2=YIP1-YI                                                        00002800 CLCT0010
      A3=XI-XINT                                                        00002810 CLCT0011
      C0=A3                                                             00002820 CLCT0012
      C1=A1-A2*AI                                                       00002830 CLCT0013
      C2=A2*(2.*AI+BI)                                                  00002840 CLCT0014
      C3=-A2*(AI+BI)                                                    00002850 CLCT0015
      AA=C2/C3                                                          00002860 CLCT0016
      BB=C1/C3                                                          00002870 CLCT0017
      CC=C0/C3                                                          00002880 CLCT0018
      A=3.*(AA/3.)**2-BB                                                00002890 CLCT0019
      B=-2.*(AA/3.)**3+BB*(AA/3.)-CC                                    00002900 CLCT0020
      PI=ACOS(-1.)                                                      00002910 CLCT0021
      P=A/3.                                                            00002920 CLCT0022
      Q=B/2.                                                            00002930 CLCT0023
      D=Q**2-P**3                                                       00002940 CLCT0024
      IF (D.GT.0.) SQD=SQRT(D)                                          00002950 CLCT0025
      E3=1./3.                                                          00002960 CLCT0026
      IF (D.GT.0.) GO TO 101                                            00002970 CLCT0027
      IF (D.EQ.0.) GO TO 102                                            00002980 CLCT0028
      IF (D.LT.0.) GO TO 103                                            00002990 CLCT0029
      GO TO 999                                                         00003000 CLCT0030
  101 IF (Q+SQD.GE.0.) XA=(Q+SQD)**E3                                   00003010 CLCT0031
      IF (Q+SQD.LT.0.) XA=-(ABS(Q+SQD))**E3                             00003020 CLCT0032
      IF (Q-SQD.GE.0.) XB=(Q-SQD)**E3                                   00003030 CLCT0033
      IF (Q-SQD.LT.0.) XB=-(ABS(Q-SQD))**E3                             00003040 CLCT0034
      X1=XA+XB-AA/3.                                                    00003050 CLCT0035
      IF (X1.GT.0..AND.X1.LT.1.) GO TO 201                              00003060 CLCT0036
```

```
      GO TO 999
 102  IF (Q.GE.0.) GO TO 502
      X1=-2.*(ABS(Q))**E3-AA/3.
      X2=-X1/2.-AA/3.
      GO TO 501
 502  X1=2.*Q**E3-AA/3.
      X2=-X1/2.-AA/3.
 501  IF (X1.GT.0..AND.X1.LT.1.) GO TO 202
 203  IF (X2.GT.0..AND.X2.LT.1.) GO TO 204
      GO TO 999
 103  U=ACOS(Q/(P**(1.5)))
      SQP=SQRT(P)
      X1=2.*SQP*COS(U/3.)-AA/3.
      X2=2.*SQP*COS(U/3.+PI*2./3.)-AA/3.
      X3=2.*SQP*COS(U/3.+PI*4./3.)-AA/3.
      IF (X1.GT.0..AND.X1.LT.1.) GO TO 205
 206  IF (X2.GT.0..AND.X2.LT.1.) GO TO 207
 208  IF (X3.GT.0..AND.X3.LT.1.) GO TO 209
      GO TO 999
 201  KFLG2=1
      RT(1)=X1
      GO TO 999
 202  KFLG2=1
      RT(1)=X1
      GO TO 203
 204  KFLG2=KFLG2+1
      RT(KFLG2)=X2
      GO TO 999
 205  KFLG2=1
      RT(1)=X1
      GO TO 206
 207  KFLG2=KFLG2+1
      RT(KFLG2)=X2
      GO TO 208
 209  KFLG2=KFLG2+1
      RT(KFLG2)=X3
```

```
999   IF (KFLG2.EQ.2.OR.KFLG2.EQ.3) WRITE (6,601)                 00003430 CLCT0073
      RETURN                                                      00003440 CLCT0074
      END                                                         00003450 CLCT0075
```

```
C  THIS SUBROUTINE TAKES A SERIES OF POINTS WITH END CONDITIONS AND      00003460 PRFR0001
C   CALLS THE APPROPRIATE FAIRING SUBROUTINE.                            00003470 PRFR0002
      SUBROUTINE PREPAR(TOL)                                             00003480 PRFR0003
      COMMON /COM2/X(30),Y(30),YORIG(30)                                 00003490 PRFR0004
      COMMON /COM6/XCRV(30),YCRV(30),YOCRV(30)                           00003500 PRFR0005
      COMMON /COM9/CRV(32,7)                                             00003510 PRFR0006
      COMMON /COM12/KCRV                                                 00003520 PRFR0007
      INTEGER BGN,END                                                    00003530 PRFR0008
      NPTS=INT(CRV(31,2)+.1)                                             00003540 PRFR0009
 1001 CONTINUE                                                           00003550 PRFR0010
      MARK=0                                                             00003560 PRFR0011
      BGN=INT(CRV(32,2))                                                 00003570 PRFR0012
      END=INT((CRV(32,2)-FLOAT(BGN)+.01)*10.)                            00003580 PRFR0013
      DO 101 I=1,NPTS                                                    00003590 PRFR0014
      KCRV=0                                                             00003600 PRFR0015
      IF (I.EQ.1.OR.INT(CRV(I,4)+.1).EQ.0.OR.INT(CRV(I,4)+.1).EQ.1) GO T00003610 PRFR0016
     10 201                                                              00003620 PRFR0017
      IF (INT(CRV(I,4)+.1).EQ.3.OR.INT(CRV(I,4)+.1).EQ.5) GO TO 401      00003630 PRFR0018
  102 KP1=K+1                                                            00003640 PRFR0019
      IF (K.EQ.1.OR.K.EQ.I) KP1=I                                        00003650 PRFR0020
      X(KP1)=CRV(I,1)                                                    00003660 PRFR0021
      Y(KP1)=CRV(I,3)                                                    00003670 PRFR0022
      YORIG(KP1)=CRV(I,2)                                                00003680 PRFR0023
      K=KP1                                                              00003690 PRFR0024
      GO TO 401                                                          00003700 PRFR0025
  101 CONTINUE                                                           00003710 PRFR0026
  999 RETURN                                                             00003720 PRFR0027
  601 EC=EC1+EC2                                                         00003730 PRFR0028
      IEC1=INT(EC1+.1)                                                   00003740 PRFR0029
      IEC2=INT((EC2+.01)*10.)                                            00003750 PRFR0030
      IF (IEC1.EQ.4.OR.IEC2.EQ.4) KCRV=1                                 00003760 PRFR0031
      IF (IEC1.EQ.4.OR.IEC2.EQ.4) CALL FARCRV(K,EC,TOL,ES1P,ES2P)        00003770 PRFR0032
      IF (KCRV.NE.1) GO TO 12                                            00003780 PRFR0033
      DO 11 JJ=1,K                                                       00003790 PRFR0034
      X(JJ)=XCRV(JJ)                                                     00003800 PRFR0035
      YORIG(JJ)=YOCRV(JJ)                                                00003810 PRFR0036
```

```
   11   Y(JJ)=YCRV(JJ)                                                   00003820 PRFR0037
   12   IF (IEC1.NE.4.AND.IEC2.NE.4)CALL FARLIN(K,EC,TOL,ES1P,ES2P)      00003830 PRFR0038
        IF (I.EQ.NPTS) GO TO 702                                         00003840 PRFR0039
        DO 701 L=1,K                                                     00003850 PRFR0040
        LC=I-L                                                           00003860 PRFR0041
        LK=K-L+1                                                         00003870 PRFR0042
        CRV(LC,1)=X(LK)                                                  00003880 PRFR0043
        CRV(LC,2)=YORIG(LK)                                              00003890 PRFR0044
  701   CRV(LC,3)=Y(LK)                                                  00003900 PRFR0045
        GO TO 101                                                        00003910 PRFR0046
  702   DO 703 L=1,K                                                     00003920 PRFR0047
        LC=I+1-L                                                         00003930 PRFR0048
        LK=K+1-L                                                         00003940 PRFR0049
        CRV(LC,1)=X(LK)                                                  00003950 PRFR0050
        CRV(LC,2)=YORIG(LK)                                              00003960 PRFR0051
  703   CRV(LC,3)=Y(LK)                                                  00003970 PRFR0052
        GO TO 101                                                        00003980 PRFR0053
  801   WRITE (6,802)                                                    00003990 PRFR0054
  802   FORMAT(' ','NOT ENOUGH DATA POINTS TO FAIR.  FAIRING BY-PASSED.') 00004000 PRFR0055
        GO TO 101                                                        00004010 PRFR0056
  201   K=I-MARK                                                         00004020 PRFR0057
        IF (K.EQ.2.AND.MARK.NE.0) GO TO 501                              00004030 PRFR0058
  202   X(K)=CRV(I,1)                                                    00004040 PRFR0059
        Y(K)=CRV(I,3)                                                    00004050 PRFR0060
        YORIG(K)=CRV(I,2)                                                00004060 PRFR0061
        IF (I.EQ.NPTS) GO TO 102                                         00004070 PRFR0062
        GO TO 101                                                        00004080 PRFR0063
  501   IM1=I-1                                                          00004090 PRFR0064
        X(1)=CRV(IM1,1)                                                  00004100 PRFR0065
        Y(1)=CRV(IM1,3)                                                  00004110 PRFR0066
        YORIG(1)=CRV(IM1,2)                                              00004120 PRFR0067
        GO TO 202                                                        00004130 PRFR0068
C  CALCULATE THE BEGINNING END CONDITIONS.                               00004140 PRFR0069
  401   MARK=I-1                                                         00004150 PRFR0070
        IF (K.LT.6) GO TO 801                                            00004160 PRFR0071
        KM1=K-1                                                          00004170 PRFR0072
```

```
      IF (I-K.EQ.1.OR.I-K.EQ.0) GO TO 301                                 00004180 PRFR0073
      IMK=I-K                                                             00004190 PRFR0074
      IF (INT(CRV(IMK,4)+.1).EQ.0.OR.INT(CRV(IMK,4)+.1).EQ.1) GO TO 302 00004200 PRFR0075
      GO TO 303                                                           00004210 PRFR0076
C  CALCULATE END CONDITIONS                                               00004220 PRFR0077
 402  IF (I.EQ.NPTS) GO TO 304                                            00004230 PRFR0078
      IF (INT(CRV(I,4)+.1).EQ.3.OR.INT(CRV(I,4)+.1).EQ.5) GO TO 305       00004240 PRFR0079
      IF (INT(CRV(I,4)+.1).EQ.0.OR.INT(CRV(I,4)+.1).EQ.1) GO TO 306       00004250 PRFR0080
      GO TO 999                                                           00004260 PRFR0081
 301  ES1P=CRV(31,3)                                                      00004270 PRFR0082
      EC1=FLOAT(BGN)                                                      00004280 PRFR0083
      GO TO 402                                                           00004290 PRFR0084
 302  EC1=2.                                                              00004300 PRFR0085
      ES1P=CRV(IMK,7)                                                     00004310 PRFR0086
      IF (INT(CRV(IMK,4)+.1).EQ.0)  EC1=3.                                00004320 PRFR0087
      GO TO 402                                                           00004330 PRFR0088
 303  IMKM1=IMK-1                                                         00004340 PRFR0089
      ES1P=ATAN((Y(IMK)-Y(IMKM1))/(X(IMK)-X(IMKM1)))                      00004350 PRFR0090
      EC1=3.                                                              00004360 PRFR0091
      GO TO 402                                                           00004370 PRFR0092
C                                                                         00004380 PRFR0093
 304  ES2P=CRV(32,3)                                                      00004390 PRFR0094
      EC2=FLOAT(END)/10.                                                  00004400 PRFR0095
      GO TO 601                                                           00004410 PRFR0096
 305  IM1=I-1                                                             00004420 PRFR0097
      ES2P=ATAN((CRV(I,3)-CRV(IM1,3))/(CRV(I,1)-CRV(IM1,1)))              00004430 PRFR0098
      EC2=.3                                                              00004440 PRFR0099
      GO TO 601                                                           00004450 PRFR0100
 306  EC2=.2                                                              00004460 PRFR0101
      GO TO 601                                                           00004470 PRFR0102
      END                                                                 00004480 PRFR0103
```

```
      SUBROUTINE FARCRV(N,EC,TOL1,ES1,ES2)                              00004490 FRCV0001
      COMMON /COM2/X(30),Y(30),YORIG(30)                                00004500 FRCV0002
      COMMON /COM5/XPRIM(30),YPRIM(30),YOPRIM(30)                       00004510 FRCV0003
      COMMON /COM6/XCRV(30),YCRV(30),YOCRV(30)                          00004520 FRCV0004
      COMMON /COM7/A(30),B(30),D(30)                                    00004530 FRCV0005
      COMMON /COM13/KCRV                                                00004540 FRCV0006
      INTEGER BGN,END                                                   00004550 FRCV0007
 1001 CONTINUE                                                          00004560 FRCV0008
      DO 10 JJ=1,N                                                      00004570 FRCV0009
      XCRV(JJ)=X(JJ)                                                    00004580 FRCV0010
      YCRV(JJ)=Y(JJ)                                                    00004590 FRCV0011
 10   YOCRV(JJ)=YORIG(JJ)                                               00004600 FRCV0012
C  R IS THE ROTATION ANGLE OF 10 DEGREES.                               00004610 FRCV0013
      PI=ACOS(-1.)                                                      00004620 FRCV0014
      R=PI/18.                                                          00004630 FRCV0015
      BGN=INT(EC)                                                       00004640 FRCV0016
      END=INT((EC-FLOAT(BGN)+.01)*10.)                                  00004650 FRCV0017
C                                                                       00004660 FRCV0018
C                                                                       00004670 FRCV0019
      ITRNS=0                                                           00004680 FRCV0020
      IFAIR=0                                                           00004690 FRCV0021
      IF (BGN.EQ.4.AND.END.NE.4) GO TO 201                              00004700 FRCV0022
      IF (END.EQ.4.AND.BGN.NE.4) GO TO 301                              00004710 FRCV0023
      GO TO 401                                                         00004720 FRCV0024
C  FAIRS CURVE FOR ONE INFINITE SLOPE AT THE BEGINNING.                 00004730 FRCV0025
 201  DO 202 I=1,6                                                      00004740 FRCV0026
      XPRIM(I)=XCRV(I)*COS(R)+YCRV(I)*SIN(R)                            00004750 FRCV0027
      YPRIM(I)=-XCRV(I)*SIN(R)+YCRV(I)*COS(R)                           00004760 FRCV0028
 202  YOPRIM(I)=-XCRV(I)*SIN(R)+YOCRV(I)*COS(R)                         00004770 FRCV0029
      ES1PRM=(4./9.)*PI                                                 00004780 FRCV0030
      TOLPRM=TOL1/COS(R)                                                00004790 FRCV0031
      ECPRM=3.1                                                         00004800 FRCV0032
      KCRV=1                                                            00004810 FRCV0033
      CALL FARLIN(6,ECPRM,TOLPRM,ES1PRM,ES2)                            00004820 FRCV0034
      DO 11 JJ=1,6                                                      00004830 FRCV0035
 11   YPRIM(JJ)=Y(JJ)                                                   00004840 FRCV0036
```

```
      DO 203 I=1,6                                                      00004850 FRCV0037
 203  YCRV(I)=XPRIM(I)*SIN(R)+YPRIM(I)*COS(R)                           00004860 FRCV0038
C  CALCULATE THE SLOPE AT POINT 3.                                      00004870 FRCV0039
      CALL SPLINE(6,4.1,ES1PRM,ES2)                                     00004880 FRCV0040
      ES1PRM=D(3)+R                                                     00004890 FRCV0041
      ES2PRM=ES2                                                        00004900 FRCV0042
      DO 204 I=3,N                                                      00004910 FRCV0043
      IDX=I-2                                                           00004920 FRCV0044
      XPRIM(IDX)=XCRV(I)                                                00004930 FRCV0045
      YPRIM(IDX)=YCRV(I)                                                00004940 FRCV0046
 204  YOPRIM(IDX)=YOCRV(I)                                              00004950 FRCV0047
      ECPRM=3.+FLOAT(END)/10.                                           00004960 FRCV0048
      NM2=N-2                                                           00004970 FRCV0049
      KCRV=1                                                            00004980 FRCV0050
      CALL FARLIN(NM2,ECPRM,TOL1,ES1PRM,ES2PRM)                         00004990 FRCV0051
      DO 12 JJ=1,NM2                                                    00005000 FRCV0052
      JJP2=JJ+2                                                         00005010 FRCV0053
      YCRV(JJP2)=Y(JJ)                                                  00005020 FRCV0054
 12   YPRIM(JJ)=Y(JJ)                                                   00005030 FRCV0055
      GO TO 500                                                         00005040 FRCV0056
C  FAIRS CURVE WHERE INFINITE SLOPE IS AT THE END.                      00005050 FRCV0057
 301  NM5=N-5                                                           00005060 FRCV0058
      NM1=N-1                                                           00005070 FRCV0059
      ECPRM=FLOAT(BGN)+.1                                               00005080 FRCV0060
      KCRV=0                                                            00005090 FRCV0061
      CALL FARLIN(NM1,ECPRM,TOL1,ES1,ES2)                               00005100 FRCV0062
      DO 13 JJ=1,NM1                                                    00005110 FRCV0063
      YCRV(JJ)=Y(JJ)                                                    00005120 FRCV0064
 13   YPRIM(JJ)=Y(JJ)                                                   00005130 FRCV0065
C  CALCULATE SLOPE AT POINT NM5.  I.E., P(NM5)                          00005140 FRCV0066
      CALL SPLINE(NM1,ECPRM,ES1,ES2)                                    00005150 FRCV0067
      ES2PRM=-(D(NM5)+R)                                                00005160 FRCV0068
      ES1PRM=(4./9.)*PI                                                 00005170 FRCV0069
      ECPRM=3.3                                                         00005180 FRCV0070
      TOLPRM=TOL1/COS(R)                                                00005190 FRCV0071
      DO 302 I=1,6                                                      00005200 FRCV0072
```

```
      IDX=N+1-I                                                         00005210 FRCV0073
      XBAR=XCRV(N)-XCRV(IDX)                                            00005220 FRCV0074
      XPRIM(I)=XBAR*COS(R)+YCRV(IDX)*SIN(R)                             00005230 FRCV0075
      YPRIM(I)=-XBAR*SIN(R)+YCRV(IDX)*COS(R)                            00005240 FRCV0076
  302 YOPRIM(I)=-XBAR*SIN(R)+YOCRV(IDX)*COS(R)                          00005250 FRCV0077
      KCRV=1                                                            00005260 FRCV0078
      CALL FARLIN(6,ECPRM,TOLPRM,ES1PRM,ES2PRM)                         00005270 FRCV0079
      DO 14 JJ=1,6                                                      00005280 FRCV0080
   14 YPRIM(JJ)=Y(JJ)                                                   00005290 FRCV0081
      DO 303 I=1.6                                                      00005300 FRCV0082
      IDX=N+1-I                                                         00005310 FRCV0083
  303 YCRV(IDX)=XPRIM(I)*SIN(R)+YPRIM(I)*COS(R)                         00005320 FRCV0084
      GO TO 500                                                         00005330 FRCV0085
C  THIS SEGMENT PAIRS THE CURVE FOR THE CASE OF INFINITE SLOPES AT BOTH 00005340 FRCV0086
  401 TOLPRM=TOL1/COS(R)                                                00005350 FRCV0087
      ECPRM=3.1                                                         00005360 FRCV0088
C  ROTATE AND PAIR THE FIRST FIVE POINTS.                               00005370 FRCV0089
      DO 402 I=1,6                                                      00005380 FRCV0090
      XPRIM(I)=XCRV(I)*COS(R)+YCRV(I)*SIN(R)                            00005390 FRCV0091
      YPRIM(I)=-XCRV(I)*SIN(R)+YCRV(I)*COS(R)                           00005400 FRCV0092
  402 YOPRIM(I)=-XCRV(I)*SIN(R)+YOCRV(I)*COS(R)                         00005410 FRCV0093
      ES1PRM=(4./9.)*PI                                                 00005420 FRCV0094
      KCRV=1                                                            00005430 FRCV0095
      CALL FARLIN(6,ECPRM,TOLPRM,ES1PRM,ES2)                            00005440 FRCV0096
      DO 15 JJ=1,6                                                      00005450 FRCV0097
   15 YPRIM(JJ)=Y(JJ)                                                   00005460 FRCV0098
C  PLACE PAIRED POINTS IN THE CRV VECTORS.                              00005470 FRCV0099
      DO 403 I=1,6                                                      00005480 FRCV0100
  403 YCRV(I)=XPRIM(I)*SIN(R)+YPRIM(I)*COS(R)                           00005490 FRCV0101
C  OBTAIN THE SLOPE AT POINT 2.                                         00005500 FRCV0102
      CALL SPLINE(6,ECPRM,ES1PRM,ES2)                                   00005510 FRCV0103
      ES2PRM=-(D(2)+2.*R)                                               00005520 FRCV0104
      ECPRM=3.3                                                         00005530 FRCV0105
C  NOW PAIR THE REMAINING POINTS ON THE LINE.                           00005540 FRCV0106
      NM1=N-1                                                           00005550 FRCV0107
      DO 404 I=1,NM1                                                    00005560 FRCV0108
```

```
      IDX=N+1-I                                                         00005570 PRCV0109
      XBAR=XCRV(N)-XCRV(IDX)                                            00005580 PRCV0110
      XPRIM(I)=XBAR*COS(R)+YCRV(IDX)*SIN(R)                             00005590 PRCV0111
      YPRIM(I)=-XBAR*SIN(R)+YCRV(IDX)*COS(R)                            00005600 PRCV0112
  404 YOPRIM(I)=-XBAR*SIN(R)+YOCRV(IDX)*COS(R)                          00005610 PRCV0113
      KCRV=1                                                            00005620 PRCV0114
      CALL PARLIN(NM1,ECPRM,TOLPRM,ES1PRM,ES2PRM)                       00005630 PRCV0115
      DO 16 JJ=1,NM1                                                    00005640 PRCV0116
   16 YPRIM(JJ)=Y(JJ)                                                   00005650 PRCV0117
      DO 405 I=1,NM1                                                    00005660 PRCV0118
      IDX=N+1-I                                                         00005670 PRCV0119
  405 YCRV(IDX)=XPRIM(I)*SIN(R)+YPRIM(I)*COS(R)                         00005680 PRCV0120
      KCRV=1                                                            00005690 PRCV0121
  500 RETURN                                                            00005700 PRCV0122
      END                                                               00005710 PRCV0123
```

```
C  THIS SUBROUTINE CALLS THE STRIP SUBROUTINES TO FAIR LINES WITHOUT      00005720 FRLN0001
C   INFINITE SLOPES.                                                      00005730 FRLN0002
      SUBROUTINE FARLIN(N,EC,TOL1,ES1,ES2)                                 00005740 FRLN0003
      COMMON /COM2/X(30),Y(30),YORIG(30)                                   00005750 FRLN0004
      COMMON /COM3/X1(5),Y1(5),YO(5)                                       00005760 FRLN0005
      COMMON /COM4/IFAIR,ACC,LIMIT                                         00005770 FRLN0006
      COMMON /COM5/XPRIM(30),YPRIM(30),YOPRIM(30)                          00005780 FRLN0007
      COMMON /COM8/Q                                                       00005790 FRLN0008
      COMMON /COM13/KCRV                                                   00005800 FRLN0009
      IF (KCRV.NE.1) GO TO 11                                              00005810 FRLN0010
      DO 10 JJ=1,N                                                         00005820 FRLN0011
      X(JJ)=XPRIM(JJ)                                                      00005830 FRLN0012
      Y(JJ)=YPRIM(JJ)                                                      00005840 FRLN0013
 10   YORIG(JJ)=YOPRIM(JJ)                                                 00005850 FRLN0014
 11   CONTINUE                                                             00005860 FRLN0015
      ITRNS=0                                                              00005870 FRLN0016
 1001 CONTINUE                                                             00005880 FRLN0017
 100  IFAIR=0                                                              00005890 FRLN0018
      NM5=N-5                                                              00005900 FRLN0019
      DO 101 I1=1,NM5                                                      00005910 FRLN0020
      IF (I1.EQ.1) CALL FSTPTS(ES1,ES2,EC,TOL1)                            00005920 FRLN0021
      IF (I1.EQ.1) GO TO 101                                               00005930 FRLN0022
      DO 102 I2=1,5                                                        00005940 FRLN0023
      INDEX=I1-1+I2                                                        00005950 FRLN0024
      X1(I2)=X(INDEX)                                                      00005960 FRLN0025
      Y1(I2)=Y(INDEX)                                                      00005970 FRLN0026
 1002 CONTINUE                                                             00005980 FRLN0027
 102  YO(I2)=YORIG(INDEX)                                                  00005990 FRLN0028
      CALL STRIP1(TOL1)                                                    00006000 FRLN0029
      I1P2=I1+2                                                            00006010 FRLN0030
      ICOUNT=0                                                             00006020 FRLN0031
      DO 103 I3=I1,I1P2                                                    00006030 FRLN0032
      ICOUNT=ICOUNT+1                                                      00006040 FRLN0033
 103  Y(I3)=Y1(ICOUNT)                                                     00006050 FRLN0034
 101  CONTINUE                                                             00006060 FRLN0035
      CALL TRANS1(N,EC,TOL1,ES1,ES2)                                       00006070 FRLN0036
```

```
      ITRNS=ITRNS+1                                                     00006080 FRLN0037
      IF (ITRNS.GE.LIMIT) GO TO 500                                     00006090 FRLN0038
      IF (IPAIR.NE.0) GO TO 100                                         00006100 FRLN0039
  500 RETURN                                                            00006110 FRLN0040
      END                                                               00006120 FRLN0041
```

```
C  THIS SUBROUTINE CALCULATES THE FAIRED POINTS FOR THE BEGINNING OF THE00006130 FSPT0001
C   LINES.                                                               00006140 FSPT0002
      SUBROUTINE FSTPTS(ES1,ES2,EC,TOL1)                                 00006150 FSPT0003
      COMMON /COM2/X(30),Y(30),YORIG(30)                                 00006160 FSPT0004
      COMMON /COM3/X1(5),Y1(5),YO(5)                                     00006170 FSPT0005
      COMMON /COM8/Q                                                     00006180 FSPT0006
      IEC=INT(EC)                                                        00006190 FSPT0007
 1001 CONTINUE                                                           00006200 FSPT0008
      IF (IEC.EQ.1) GO TO 301                                            00006210 FSPT0009
      IF (IEC.EQ.2) GO TO 302                                            00006220 FSPT0010
      IF (IEC.EQ.3) GO TO 303                                            00006230 FSPT0011
C  THIS LOOP ESTABLISHES THE FIRST THREE POINTS FOR FREE ENDS.           00006240 FSPT0012
 301  DO 401  I2=1,5                                                     00006250 FSPT0013
      Y1(I2)=Y(I2)                                                       00006260 FSPT0014
      X1(I2)=X(I2)                                                       00006270 FSPT0015
      X1(I1)=X(I2)                                                       00006280 FSPT0016
 401  YO(I2)=YORIG(I2)                                                   00006290 FSPT0017
      CALL STRIP1(TOL1)                                                  00006300 FSPT0018
      DO 402 I3=1,3                                                      00006310 FSPT0019
 402  Y(I3)=Y1(I3)                                                       00006320 FSPT0020
      GO TO 101                                                          00006330 FSPT0021
C  THIS LOOP ESTABLISHES THE SECOND AND THIRD POINTS FOR FIXED ENDS.     00006340 FSPT0022
 302  DO 403 I4=1,5                                                      00006350 FSPT0023
      X1(I4)=X(I4)                                                       00006360 FSPT0024
      Y1(I4)=Y(I4)                                                       00006370 FSPT0025
 403  YO(I4)=YORIG(I4)                                                   00006380 FSPT0026
      CALL STRIP2(TOL1)                                                  00006390 FSPT0027
      Y(2)=Y1(2)                                                         00006400 FSPT0028
      Y(3)=Y1(3)                                                         00006410 FSPT0029
      GO TO 101                                                          00006420 FSPT0030
C  THIS LOOP ESTABLISHES THE SECOND AND THIRD POINTS WITH SLOPE GIVEN.   00006430 FSPT0031
 303  DO 405 I6=1,4                                                      00006440 FSPT0032
      X1(I6)=X(I6)                                                       00006450 FSPT0033
      Y1(I6)=Y(I6)                                                       00006460 FSPT0034
 405  YO(I6)=YORIG(I6)                                                   00006470 FSPT0035
      Q=ES1                                                              00006480 FSPT0036
```

```
      CALL STRIP3(TOL1)                                            00006490 FSPT0037
      Y(2)=Y1(2)                                                   00006500 FSPT0038
      DO 406 I7=2,6                                                00006510 FSPT0039
      I7M1=I7-1                                                    00006520 FSPT0040
      X1(I7M1)=X(I7)                                               00006530 FSPT0041
      Y1(I7M1)=Y(I7)                                               00006540 FSPT0042
  406 YO(I7M1)=YORIG(I7)                                           00006550 FSPT0043
      CALL STRIP1(TOL1)                                            00006560 FSPT0044
      Y(3)=Y1(2)                                                   00006570 FSPT0045
  101 RETURN                                                       00006580 FSPT0046
      END                                                          00006590 FSPT0047
```

```
C  THIS SUBROUTINE DHANGES THE ABCISSAS OF THE END POINTS TO ORIENT THE 00006600 TRNS0001
C   POINTS AS IF THEY ARE AT THE ORIGIN.                                 00006610 TRNS0002
      SUBROUTINE TRANS1(N,EC,TOL1,ES1,ES2)                               00006620 TRNS0003
      COMMON /COM1/SMAT(4,4),T(4),WKAREA(4)                              00006630 TRNS0004
      COMMON /COM2/X(30),Y(30),YORIG(30)                                 00006640 TRNS0005
      COMMON /COM3/X1(5),Y1(5),YO(5)                                     00006650 TRNS0006
      COMMON /COM4/IPAIR,ACC,LIMIT                                       00006660 TRNS0007
      INTEGER DEC                                                        00006670 TRNS0008
 1001 CONTINUE                                                           00006680 TRNS0009
      NM1=N-1                                                            00006690 TRNS0010
      NM2=N-2                                                            00006700 TRNS0011
      DO 101 I1=1,5                                                      00006710 TRNS0012
      NMI1P1=N-I1+1                                                      00006720 TRNS0013
      X1(I1)=X(N)-X(NMI1P1)                                              00006730 TRNS0014
      Y1(I1)=Y(NMI1P1)                                                   00006740 TRNS0015
 101  YO(I1)=YORIG(NMI1P1)                                               00006750 TRNS0016
      DEC=INT((EC-FLOAT(INT(EC))+.01)*10.)                               00006760 TRNS0017
      IF (DEC.EQ.1) GO TO 301                                            00006770 TRNS0018
      IF (DEC.EQ.2) GO TO 302                                            00006780 TRNS0019
      IF (DEC.EQ.3) GO TO 303                                            00006790 TRNS0020
      Y(1)=9999.                                                         00006800 TRNS0021
      GO TO 500                                                          00006810 TRNS0022
C                                                                        00006820 TRNS0023
 301  CALL STRIP1(TOL1)                                                  00006830 TRNS0024
      Y(N)=Y1(1)                                                         00006840 TRNS0025
      Y(NM1)=Y1(2)                                                       00006850 TRNS0026
      Y(NM2)=Y1(3)                                                       00006860 TRNS0027
      GO TO 500                                                          00006870 TRNS0028
C                                                                        00006880 TRNS0029
 302  CALL STRIP2(TOL1)                                                  00006890 TRNS0030
      Y(NM1)=Y1(2)                                                       00006900 TRNS0031
      Y(NM2)=Y1(3)                                                       00006910 TRNS0032
      GO TO 500                                                          00006920 TRNS0033
C                                                                        00006930 TRNS0034
 303  Q=-ES2                                                             00006940 TRNS0035
      CALL STRIP3(TOL1)                                                  00006950 TRNS0036
```

```
      Y(NM1)=Y1(2)                                                      00006960 TRNS0037
      DO 304 I2=1,5                                                     00006970 TRNS0038
      NMI2=N-I2                                                         00006980 TRNS0039
      X1(I2)=X(NMI2)                                                    00006990 TRNS0040
      Y1(I2)=Y(NMI2)                                                    00007000 TRNS0041
  304 YO(I2)=YORIG(NMI2)                                                00007010 TRNS0042
      CALL STRIP1(TOL1)                                                 00007020 TRNS0043
      Y(NM2)=Y1(2)                                                      00007030 TRNS0044
  500 RETURN                                                            00007040 TRNS0045
      END                                                               00007050 TRNS0046
```

```
C  THIS SUBROUTINE CALCULATES THE FIRST THREE PAIRED POINTS (OR CENTER  00007060 STP10001
C   POINT) OF A SERIES OF FIVE DATA POINTS.  STRIP1 WOULD BE SUBSEQUENTL00007070 STP10002
C   APPLIED TO CONTINUE THE PAIRING PROCESS.                            00007080 STP10003
      SUBROUTINE STRIP1(TOL1)                                           00007090 STP10004
      COMMON /COM1/SMAT(4,4),T(4),WKAREA(4)                             00007100 STP10005
      COMMON /COM3/X1(5),Y1(5),YO(5)                                    00007110 STP10006
      COMMON /COM4/IPAIR,ACC,LIMIT                                      00007120 STP10007
 1001 CONTINUE                                                          00007130 STP10008
      SMAT(1,1)=5.                                                      00007140 STP10009
      DO 101 I1=1,6                                                     00007150 STP10010
      SI1=0.                                                            00007160 STP10011
      DO 102 I2=1,5                                                     00007170 STP10012
 102  SI1=SI1+X1(I2)**I1                                                00007180 STP10013
      I1P1=I1+1                                                         00007190 STP10014
      IF (I1.LE.3) GO TO 103                                            00007200 STP10015
      I1M2=I1-2                                                         00007210 STP10016
      IF (I1.GE.4) GO TO 104                                            00007220 STP10017
C                                                                       00007230 STP10018
 101  CONTINUE                                                          00007240 STP10019
      GO TO 105                                                         00007250 STP10020
 103  SMAT(1,I1P1)=SI1                                                  00007260 STP10021
      GO TO 101                                                         00007270 STP10022
 104  SMAT(4,I1M2)=SI1                                                  00007280 STP10023
      GO TO 101                                                         00007290 STP10024
 105  CONTINUE                                                          00007300 STP10025
      SMAT(2,1)=SMAT(1,2)                                               00007310 STP10026
      SMAT(3,1)=SMAT(1,3)                                               00007320 STP10027
      SMAT(2,2)=SMAT(1,3)                                               00007330 STP10028
      SMAT(4,1)=SMAT(1,4)                                               00007340 STP10029
      SMAT(3,2)=SMAT(1,4)                                               00007350 STP10030
      SMAT(2,3)=SMAT(1,4)                                               00007360 STP10031
      SMAT(3,3)=SMAT(4,2)                                               00007370 STP10032
      SMAT(2,4)=SMAT(4,2)                                               00007380 STP10033
      SMAT(3,4)=SMAT(4,3)                                               00007390 STP10034
 1002 CONTINUE                                                          00007400 STP10035
C                                                                       00007410 STP10036
```

```
      DO 106 I3=1,4                                                     00007420 STP10037
      TI3=0.                                                            00007430 STP10038
      DO 107 I4=1,5                                                     00007440 STP10039
      IF (X1(I4).EQ.0..AND.I3.EQ.1) TI3=TI3+Y1(I4)                      00007450 STP10040
  107 IF (X1(I4).NE.0..OR.I3.NE.1) TI3=TI3+Y1(I4)*X1(I4)**(I3-1)        00007460 STP10041
  106 T(I3)=TI3                                                         00007470 STP10042
C                                                                       00007480 STP10043
C  NOW CALL LEQT1F FROM THE IMSL LIBRARY TO CALCULATE THE CUBIC         00007490 STP10044
C    COEFFICIENTS.                                                      00007500 STP10045
      CALL LEQT1F(SMAT,1,4,4,T,0,WKAREA,IER)                            00007510 STP10046
      YP1=T(1)+T(2)*X1(1)+T(3)*X1(1)**2+T(4)*X1(1)**3                   00007520 STP10047
      YP2=T(1)+T(2)*X1(2)+T(3)*X1(2)**2+T(4)*X1(2)**3                   00007530 STP10048
      YP3=T(1)+T(2)*X1(3)+T(3)*X1(3)**2+T(4)*X1(3)**3                   00007540 STP10049
      DELTA1=YP1-YO(1)                                                  00007550 STP10050
      DELTA=ABS(DELTA1)                                                 00007560 STP10051
      IF (DELTA.GE.TOL1) Y1(1)=YO(1)+SIGN(TOL1,DELTA1)                  00007570 STP10052
      IF (ABS(Y1(1)-YP1).GT.ACC) IFAIR=IFAIR+1                          00007580 STP10053
      IF (DELTA.LT.TOL1) Y1(1)=YP1                                      00007590 STP10054
      DELTA1=YP2-YO(2)                                                  00007600 STP10055
      DELTA=ABS(DELTA1)                                                 00007610 STP10056
      IF (DELTA.GE.TOL1) Y1(2)=YO(2)+SIGN(TOL1,DELTA1)                  00007620 STP10057
      IF (ABS(Y1(2)-YP2).GT.ACC) IFAIR=IFAIR+1                          00007630 STP10058
      IF (DELTA.LT.TOL1) Y1(2)=YP2                                      00007640 STP10059
      DELTA1=YP3-YO(3)                                                  00007650 STP10060
      DELTA=ABS(DELTA1)                                                 00007660 STP10061
      IF (DELTA.GE.TOL1) Y1(3)=YO(3)+SIGN(TOL1,DELTA1)                  00007670 STP10062
      IF (ABS(Y1(3)-YP3).GT.ACC) IFAIR=IFAIR+1                          00007680 STP10063
      IF (DELTA.LT.TOL1) Y1(3)=YP3                                      00007690 STP10064
      RETURN                                                            00007700 STP10065
      END                                                               00007710 STP10066
```

```
C  THIS SUBROUTINE CALCULATES THE SECOND AND THIRD FAIRED POINTS FROM    00007720 STP20001
C   A SERIES OF FIVE DATA POINTS WITH END POINT FIXED.  STRIP1 WOULD BE 00007730 STP20002
C   SUBSEQUENTLY APPLIED TO CONTINUE THE FAIRING PROCESS.               00007740 STP20003
      SUBROUTINE STRIP2(TOL1)                                           00007750 STP20004
      COMMON /COM1/SMAT(4,4),T(4),WKAREA(4)                             00007760 STP20005
      COMMON /COM3/X1(5),Y1(5),YO(5)                                    00007770 STP20006
      COMMON /COM4/IFAIR,ACC,LIMIT                                      00007780 STP20007
      XREF=X1(1)                                                        00007790 STP20008
      DO 91 L1=1,4                                                      00007800 STP20009
 91   X1(L1)=X1(L1)-XREF                                                00007810 STP20010
      DO 101 I1=2,6                                                     00007820 STP20011
      I1M1=I1-1                                                         00007830 STP20012
      SI1=0.                                                            00007840 STP20013
      DO 102 I2=2,5                                                     00007850 STP20014
 102  SI1=SI1+X1(I2)**I1                                                00007860 STP20015
      IF (I1M1.LE.3) GO TO 103                                          00007870 STP20016
      I1M2=I1-2                                                         00007880 STP20017
      IF (I1M1.GE.4) GO TO 104                                          00007890 STP20018
 101  CONTINUE                                                          00007900 STP20019
      GO TO 105                                                         00007910 STP20020
 103  SMAT(1,I1M1)=SI1                                                  00007920 STP20021
      GO TO 101                                                         00007930 STP20022
C                                                                       00007940 STP20023
 104  SMAT(3,I1M2)=SI1                                                  00007950 STP20024
      GO TO 101                                                         00007960 STP20025
 105  CONTINUE                                                          00007970 STP20026
      SMAT(2,1)=SMAT(1,2)                                               00007980 STP20027
      SMAT(3,1)=SMAT(1,3)                                               00007990 STP20028
      SMAT(2,2)=SMAT(1,3)                                               00008000 STP20029
      SMAT(2,3)=SMAT(3,2)                                               00008010 STP20030
C                                                                       00008020 STP20031
      DO 106 I3=1,3                                                     00008030 STP20032
      TI3=0.                                                            00008040 STP20033
      DO 107 I4=2,5                                                     00008050 STP20034
 107  TI3=TI3+(Y1(I4)-Y1(1))*X1(I4)**I3                                 00008060 STP20035
 106  T(I3)=TI3                                                         00008070 STP20036
```

```
C  NOW CALL LEQT1F FROM THE IMSL LIBRARY TO CALCULATE THE COEFFICIENTS. 00008080 STP20037
      CALL LEQT1F(SMAT,1,3,4,T,0,WKAREA,IER)                             00008090 STP20038
      YP2=Y1(1)+T(1)*X1(2)+T(2)*X1(2)**2+T(3)*X1(2)**3                   00008100 STP20039
      YP3=Y1(1)+T(1)*X1(3)+T(2)*X1(3)**2+T(3)*X1(3)**3                   00008110 STP20040
      DELTA1=YP2-YO(2)                                                   00008120 STP20041
      DELTA=ABS(DELTA1)                                                  00008130 STP20042
      IF (DELTA.GE.TOL1) Y1(2)=YO(2)+SIGN(TOL1,DELTA1)                   00008140 STP20043
      IF (ABS(Y1(2)-YP2).GT.ACC) IFAIR=IFAIR+1                           00008150 STP20044
      IF (DELTA.LT.TOL1) Y1(2)=YP2                                       00008160 STP20045
      DELTA1=YP3-YO(3)                                                   00008170 STP20046
      DELTA=ABS(DELTA1)                                                  00008180 STP20047
      IF (DELTA.GE.TOL1) Y1(3)=YO(3)+SIGN(TOL1,DELTA1)                   00008190 STP20048
      IF (ABS(Y1(3)-YP3).GT.ACC) IFAIR=IFAIR+1                           00008200 STP20049
      IF (DELTA.LT.TOL1) Y1(3)=YP3                                       00008210 STP20050
      DO 92 L2=1,4                                                       00008220 STP20051
   92 X1(L2)=X1(L2)+XREF                                                 00008230 STP20052
      RETURN                                                             00008240 STP20053
      END                                                                00008250 STP20054
```

```
C  THIS SUBROUTINE CALCULATES THE FAIRED SECOND DATA POINT FOR A SERIES 00008260 STP30001
C   OF FOUR DATA POINTS.  STRIP1 WOULD BE SUBSEQUENTLY APPLIED TO       00008270 STP30002
C   CONTINUE THE FAIRING PROCESS.                                       00008280 STP30003
      SUBROUTINE STRIP3(TOL1)                                           00008290 STP30004
      COMMON /COM3/X1(5),Y1(5),YO(5)                                    00008300 STP30005
      COMMON /COM4/IFAIR,ACC,LIMIT                                      00008310 STP30006
      COMMON /COM8/Q                                                    00008320 STP30007
      XREF=X1(1)                                                        00008330 STP30008
      DO 91 L1=1,4                                                      00008340 STP30009
 91   X1(L1)=X1(L1)-XREF                                                00008350 STP30010
      DO 101 I1=4,6                                                     00008360 STP30011
      SI1=0.                                                            00008370 STP30012
      DO 102 I2=2,4                                                     00008380 STP30013
 102  SI1=SI1+X1(I2)**I1                                                00008390 STP30014
      IF (I1.EQ.4) S4=SI1                                               00008400 STP30015
      IF (I1.EQ.5) S5=SI1                                               00008410 STP30016
 101  IF (I1.EQ.6) S6=SI1                                               00008420 STP30017
      DO 103 I3=2,3                                                     00008430 STP30018
      TI3=0.                                                            00008440 STP30019
      DO 104 I4=2,4                                                     00008450 STP30020
 104  TI3=TI3+(Y1(I4)-Q*X1(I4)-Y1(1))*X1(I4)**I3                        00008460 STP30021
      IF (I3.EQ.2) T2=TI3                                               00008470 STP30022
 103  IF (I3.EQ.3) T3=TI3                                               00008480 STP30023
      A2=(T2*S6-T3*S5)/(S4*S6-S5**2)                                    00008490 STP30024
      A3=(T3*S4-T2*S5)/(S4*S6-S5**2)                                    00008500 STP30025
      YP2=Y1(1)+Q*X1(2)+A2*X1(2)**2+A3*X1(2)**3                         00008510 STP30026
      DELTA1=YP2-YO(2)                                                  00008520 STP30027
      DELTA=ABS(DELTA1)                                                 00008530 STP30028
      IF (DELTA.GE.TOL1) Y1(2)=YO(2)+SIGN(TOL1,DELTA1)                  00008540 STP30029
      IF (ABS(Y1(2)-YP2).GT.ACC) IFAIR=IFAIR+1                          00008550 STP30030
      IF (DELTA.LT.TOL1) Y1(2)=YP2                                      00008560 STP30031
      DO 92 L2=1,4                                                      00008570 STP30032
 92   X1(L2)=X1(L2)+XREF                                                00008580 STP30033
      RETURN                                                            00008590 STP30034
      END                                                               00008600 STP30035
```

APPENDIX G

The following is an example of a typical bow section for a ship with a bulbous bow mounted sonar dome. Although the section is for a hypothetical ship it illustrates the capability of the computer program to fair, spline and interpolate a curve having an infinite slope at an end point.

In the CRV matrix shown, the items written in block numbers are input values while those in italics are values calculated by the program. The elements left blank were not used in this example. The values of tolerence, accuracy and limit are:

TOL = 1.00

ACC = 0.05

LIMIT = 10

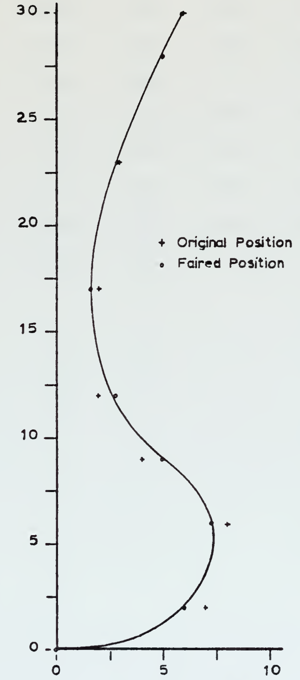


Figure G.1 – Bow Section (Bulbous)

CRV MATRIX

	1	2	3	4	5	6	7
1	0.0	0.0	0.000	0.0	0.333	-0.480	1.570
2	2.0	7.0	6.000	0.0	0.530	-0.675	0.802
3	6.0	8.0	7.301	0.0	0.394	-0.122	-0.279
4	9.0	4.0	4.999	0.0	-0.156	0.160	-0.776
5	12.0	2.0	2.805	0.0	-0.242	0.241	-0.473
6	17.0	2.0	1.609	0.0	-0.218	0.149	0.002
7	23.0	3.0	2.934	0.0	-0.041	0.009	0.366
8	28.0	5.0	5.086	0.0	0.006	-0.003	0.416
9	30.0	6.0	5.954	0.0			0.407
⋮							
31	1.0	9.0	0.0	1.1			
32	2.0	4.1	0.0				

The resulting interpolated values for increments of $\Delta X = (30-0)/20$ are as follows:

X	Y	DY/DX
0.0	0.0000	******
1.5	5.4206	1.3048
3.0	6.8443	0.6555
4.5	7.4269	0.1361
6.0	7.3018	-0.2867
7.5	6.4713	-0.8319
9.0	4.9988	-0.9816
10.5	3.7259	-0.7238
12.0	2.8047	-0.5114
13.5	2.1714	-0.3371
15.0	1.7846	-0.1815
16.5	1.6191	-0.0415
18.0	1.6542	0.0867
19.5	1.8721	0.2010
21.0	2.2473	0.2956
22.5	2.7465	0.3656
24.0	3.3316	0.4116
25.5	3.9724	0.4396
27.0	4.6403	0.4476
28.5	5.3051	0.4368
30.0	5.9543	0.4308

Figure G.1 shows the curve generated as a result of fairing the given data points.

Thesis 190199
H1038 Haberlandt
c.1 Computer aided
geometrical variation
and fairing of ship
hull forms.

Thesis 190199
H1038 Haberlandt
c.1 Computer aided
geometrical variation
and fairing of ship
hull forms.

thesH1038
Computer aided geometrical variation and
3 2768 002 13625 1
DUDLEY KNOX LIBRARY